Proceedings of the Conference on Computational Engineering and Science JSCES
Vol. 30, Jun. 2025

S-version FEM-based strategy for predicting high-speed crack propagation/arrest behaviour in 3D cross-joint structures

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This study introduces a strategy utilising the s-version finite element method to predict high-speed crack propagation and arrest phenomena in 3D cross-joint structures. Material resistance was determined through generation phase analysis, based on experiments with plate structures made of polymethyl methacrylate (PMMA). An application phase analysis methodology was then developed, incorporating nonlinear gradient descent and golden section techniques. The effectiveness of the model was validated by a comparative analysis of experimental and predictive results.

Key Words: *S-version of the finite element method, Fracture mechanics, cross-joint structure, 3D high-speed crack propagation prediction*

1. INTRODUCTION

Numerical methodologies which can accurately and efficiently predicting the high-speed crack propagation in cross-joint structures is essential for ensuring the safety of the large structures.

This study introduces a strategy utilising the s-version finite element method (the s-method) for application phase analysis, aimed at predicting high-speed crack propagation and arrest behaviours in 3D structures. The s-method employs a coarse global mesh to reduce computational costs while maintaining a detailed local mesh around the crack front to enhance accuracy [1].

The proposed strategy is divided into two segments: the first predicts the crack behaviour post-initiation, and the second addresses the subsequent high-speed crack propagation and arrest behaviours. A novel optimisation algorithm that incorporates nonlinear gradient descent and golden section methods is implemented in both segments to refine these predictions.

Validation of this strategy was achieved through experiments with double cantilever beam-type specimens made of polymethyl methacrylate (PMMA). Initial tests, building upon previous research [2], identified material resistance using generation phase analysis of plate PMMA specimens. This resistance served as input for subsequent predictions. Each test's predictions were juxtaposed with experimental data, confirming the strategy's efficacy in reproducing the dynamic crack behaviours, including crack velocity oscillations likely induced by specimen vibrations. These results underscore the potential of the proposed strategy as a foundational numerical framework for predicting high-speed crack propagation and arrest behaviours in engineering structures.

The remainder of this paper is organised as follows. Section 2 introduces the modelling approach based on the s-method for simulating high-speed crack propagation in 3D cross-joint structures, including the approach for evaluating the dynamic stress intensity factor (DSIF). In Section 3 we elucidate the proposed strategy for application phase analysis, focused on predicting high-speed crack propagation behaviours in cross-joint structures. The results of our proposed strategy are presented in Section 4. Finally, Section 5 summaries the conclusions derived from this study.

2. MODELLING 3D DYNAMIC CRACK PROPAGATION

As a prerequisite to predict the crack propagation/arrest behaviour, this section introduces an approach for modelling high-speed crack propagation in a cross-joint structure. The proposed approach is based on the s-method, ensuring both accuracy and efficiency in the modelling process. Additionally, the assessment of well-established fracture parameters, specifically the dynamic stress intensity factor (DSIF), is incorporated to provide a quantitative characterisation of high-speed crack propagation.

(1) Fundamentals of the s-version FEM

The s-method is an advanced numerical technique that employs multiple meshes superimposed within a finite element framework. This method was originally proposed by Fish [3]. In this study, two types of meshes are used: global mesh and local mesh. Each mesh used in this study consisted of eight-node hexahedral elements to simplify the analysis. The global mesh is a relatively coarse mesh representing the entire problem domain Ω^{G}, and the local mesh is a finer mesh representing the local domain Ω^{L}. The Dirichlet and Neumann boundary conditions are imposed on the boundaries Γ_u and Γ_t, respectively. A schematic illustrating the global and local meshes employed in the s-method, along with their boundary conditions, is depicted in **Fig. 1**.

The displacement vector $\mathbf{u}(\mathbf{x})$ and acceleration vector $\ddot{\mathbf{u}}(\mathbf{x})$ at the coordinate $\mathbf{x}$ are expressed as:

$$\mathbf{u}(\mathbf{x}) = \begin{cases} \mathbf{u}^{\mathrm{G}}(\mathbf{x}) & \text{in } \Omega^{\mathrm{G}} \setminus \Omega^{\mathrm{L}} \\ \mathbf{u}^{\mathrm{G}}(\mathbf{x}) + \mathbf{u}^{\mathrm{L}}(\mathbf{x}) & \text{in } \Omega^{\mathrm{L}} \end{cases}, \tag{1}$$

$$\ddot{\mathbf{u}}(\mathbf{x}) = \begin{cases} \ddot{\mathbf{u}}^{\mathrm{G}}(\mathbf{x}) & \text{in } \Omega^{\mathrm{G}} \setminus \Omega^{\mathrm{L}} \\ \ddot{\mathbf{u}}^{\mathrm{G}}(\mathbf{x}) + \ddot{\mathbf{u}}^{\mathrm{L}}(\mathbf{x}) & \text{in } \Omega^{\mathrm{L}} \end{cases}, \tag{2}$$

where $\mathbf{u}^{\mathrm{G}}(\mathbf{x})$ and $\mathbf{u}^{\mathrm{L}}(\mathbf{x})$ are the displacement components corresponding to the global and local meshes, respectively, while $\ddot{\mathbf{u}}^{\mathrm{G}}(\mathbf{x})$ and $\ddot{\mathbf{u}}^{\mathrm{L}}(\mathbf{x})$ are the acceleration components corresponding to the global and local meshes, respectively. Eqs. (1) and (2) illustrate that the displacement and acceleration fields within the local domain Ω^{L} are defined as a "superposition" of the global and local components. To ensure the continuity of the displacement $\mathbf{u}(\mathbf{x})$ on the boundary of the local domain, Γ^{L}, the following Dirichlet boundary condition is applied:

$$\mathbf{u}^{\mathrm{L}}(\mathbf{x}) = \mathbf{0} \quad \text{on } \Gamma^{\mathrm{L}}. \tag{3}$$

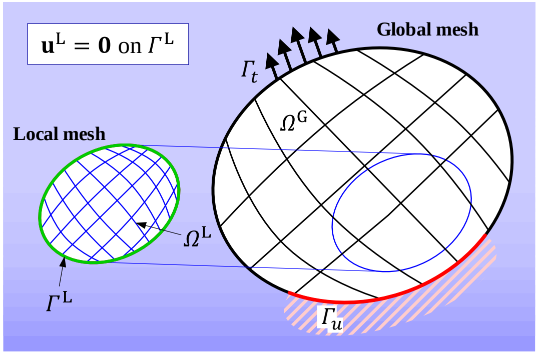


Fig. 1. Global and local meshes and their boundary conditions defined in the s-method.

(2) Crack representation

This study aims to predict the behaviour of high-speed planar crack propagation in a cross-joint structure under tensile stress (i.e. a Mode I crack), which is the most common and significant scenario in practical brittle crack phenomena. Considering that the shape of the crack front varies over time, simulating the transition of the crack front shape is a crucial aspect of the proposed strategy.

Fig. 2 illustrates a schematic visualisation of high-speed crack propagation in a cross-joint structure under tensile stress (Mode I) at different time steps, with a focus on the crack-front shape on the xz-symmetric plane. For simplicity in analysis, we assume that the crack-front shape is symmetric in the thickness direction, which allowing us to focus on merely one-quarter of the entire cross-joint structure. The crack front exhibits a curved shape, and its shape is approximated by the following function:

$$x(z, x_{\mathrm{tip}}) = A_6(x_{\mathrm{tip}}) z^6 + A_2(x_{\mathrm{tip}}) z^2 + x_{\mathrm{tip}}. \tag{4}$$

where x_{tip} denotes the x-coordinate of the deepest point of the crack front which located at the mid-thickness of the structure.

For the modelling of high-speed crack propagation, we implemented the nodal force release technique [1] on the local mesh. In each step, a linear nodal force release is assumed, whereby the nodal force at the crack front nodes varies with time. The local mesh is updated to accommodate high-speed crack propagation, allowing for flexible adjustments. The local mesh is updated according to the dynamic crack propagation by utilising its flexible definition. For a more comprehensive understanding of the nodal force release technique and local mesh update procedure, please refer to [4].

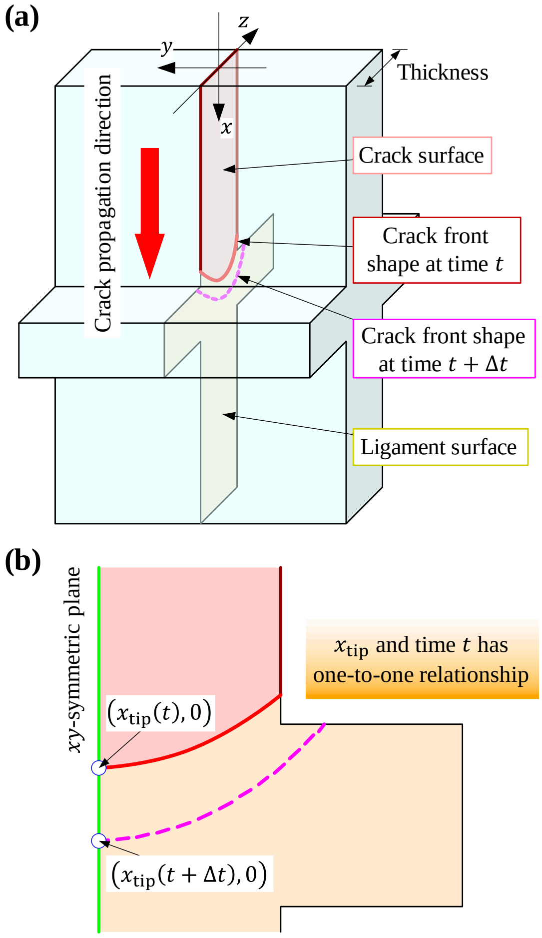


Fig. 2. High-speed crack propagation in a cross-joint structure at different time steps.

(3) Evaluation of DSIF

High-speed crack propagation and arrest in elastic materials are typically characterised using the dynamic stress intensity factor (DSIF), K_{I}, or the dynamic J-integral. In this study, we assess the dynamic J-integral through the application of Gauss's divergence theorem to obtain the domain integral form. For more comprehensive details, please refer to [4].

3. PROPOSE STRATGEY FOR APPLICATION PHASE ANALYSIS

Specifically, we will introduce the material resistance and the crack arrest model, enabling us to assess the crack propagation status during prediction. Furthermore, given the inverse nature of the application phase analysis, we propose an optimisation algorithm for predicting crack propagation behaviours. Building upon the s-method-based dynamic crack propagation model presented in the previous section, this section introduces a detailed presentation of the proposed strategy for the application phase analysis. This strategy is designed to predict high-speed crack propagation/arrest behaviour in 3D structures, which has not been established in past works. Specifically, we will introduce the crack propagation and arrest conditions, enabling us to assess the crack propagation status during prediction. Furthermore, given the inverse nature of the application phase analysis, we propose an optimisation algorithm for predicting crack propagation behaviours.

(1) Crack propagation and arrest conditions

In Section 1, it was previously emphasised that the application phase analysis operates in an inverse manner, with its input data derived from the output results of the forward analysis, namely the generation phase analysis. Hence, the pivotal output derived from the generation phase analysis, the material resistance, serves as the judgement of crack propagation for the application phase analysis. The choice of the material resistance as the input is founded on its status as the most comprehensive and representative outcome of the generation phase analysis, offering a quantitative gauge of the intricacies of crack propagation. Additionally, we have introduced a specialised crack arrest model aimed at precisely predicting the position of crack arrest. Subsequent paragraphs will provide a detailed explanation of the material resistance and crack arrest model.

Given that the focus of this study is on elastic, isotropic, homogeneous structures, it is pertinent to note that the DSIF serves as a standard metric for characterising dynamic crack propagation behaviours [1]. Furthermore, the relationship between DSIF and crack velocity is frequently employed as the criterion to predict crack propagation behaviours in application phase analysis. Therefore, in this study, we establish the DSIF as the cornerstone of the material resistance. In this study, the relationship between DSIF and crack velocity, denoted as $K_{\mathrm{Id}}(V)$, for plate structure is utilised as the material resistance to predict the crack propagation in cross-joint structure.

The concept of crack arrest signifies the termination of high-speed crack propagation upon the fulfilment of specific conditions. In this study, the crack arrest condition is defined as the absence of a solution that provides a positive crack velocity along the crack front. Given that the crack-front shape is approximated using a quadratic function, the assessment of crack arrest is simplified by focusing on the crack velocity at two critical locations: (i) the mid-thickness and (ii) the surface of structure. Crack arrest is deemed to occur if either (i) the crack velocity at the mid-thickness or (ii) the crack velocity at the surface of structure becomes negative.

(2) Optimisation algorithm for application phase analysis

Within the proposed optimisation algorithm, we focus on two critical parameters characterising crack propagation behaviours: the crack velocity at mid-thickness, denoted as V, and the parameter characterising the crack-front shape, A_6, A_2. Consequently, the optimisation process seeks to determine the optimal values for these key parameters based on the provided input data.

The objective of the proposed strategy is to predict the high-speed crack propagation/arrest behaviour under the specified loading condition. Specifically, we aim to establish a one-to-one relationship between the crack arrest length and these conditions. Consequently, predictions of the crack initiation are excluded from this analysis. The reason is that the output predictions from models based on the widely accepted weakest-link assumption for crack initiation typically manifest as either the probability of failure under specified loads or the distribution of loads at which failure occurs. The inclusion of crack initiation predictions, which would yield a distribution of crack arrest lengths, fundamentally contradicts our goal of establishing a direct correlation between crack arrest length and loading condition.

As a result of excluding crack initiation, the application phase analysis is divided into two segments:

- Segment I concentrates on simulating the stable crack propagation behaviour after crack initiation under the specified loading condition. The fulfilling the crack propagation condition signifies the completion of this segment. Since this segment focuses solely the end of the crack initiation stage, evaluating crack arrest based on the crack arrest condition is not required.
- Segment II aims to simulate the following overall high-speed crack propagation behaviour. This segment consists of multiple discretised prediction steps, with the completion of each step assessed through the fulfilment of the crack propagation condition. In addition, the crack arrest condition is employed at each step to determine the occurrence of arrest.

As previously mentioned in Section 3.1, we incorporate the material resistance into the objective function in optimisation algorithm. The objective function for the optimisation is defined as follows:

$$f(\bar{V},\overline{A_2}) = \log_{10}\sum_{i=1}^{k}\left(K_{\mathrm{I}}^{i}(\bar{V}\times V^{\mathrm{const}},\overline{A_6}\times A_6^{\mathrm{const}},\overline{A_2}\times A_2^{\mathrm{const}}) - K_{\mathrm{Id}}(V^i)\right)^2, \quad (5)$$

where k denotes the number of local nodes on the crack front. K_{I}^{i} represents the DSIF at local node i, evaluated through the generation phase analysis with the numerical model generated using input parameters. K_{Id} is the expression for the material resistance discussed in Section 3.1. Additionally, V^i signifies the crack velocity at local node i.

4. VALIDATIONS AND DISCUSSIONS

To validate the proposed strategy, we conducted a two-step analysis. Initially, we performed generation phase analysis, which is based on experimental results and the methodologies presented in Sections 2. The material resistance $K_{\mathrm{Id}}(V)$ is derived based on the numerical results obtained from generation phase analysis. Subsequently, employing the derived material resistance, we performed the application phase analysis based on our proposed strategy to predict the high-speed crack propagation behaviours. The predicted results were then thoroughly compared with the experimental data.

Table. 1. Summary of experimental results.

Specimen	Notch radius R [mm]	Clip gauge opening displacement δ_{clip} [mm]	Crack arrest location x_{a} [mm]
J1	1.0	0.750	53.75
J2	2.2	0.729	52.77

(1) Target experiments

To capture the dynamic transition of the crack front shape in a cross-joint structure, we conducted target experiments using PMMA as the material [5]. We performed three experiments using double cantilever beam (DCB)-type specimens, which are commonly used in crack arrest tests [5]. The experimental setup is illustrated in **Fig. 3**, and the experimental results for the clip gauge opening displacement, δ_{clip}, and crack arrest length, x_{a}, which is defined as the deepest point on the crack front, are summarised in Table 1. The results of the crack-front approximation corresponding using Eq. (4) based on experimental results are displayed in **Fig.4**.

(2) Numerical results

Based on the experimental results and the proposed strategy, the generation phase analysis is conducted to simulate the crack propagation in cross-joint structures.

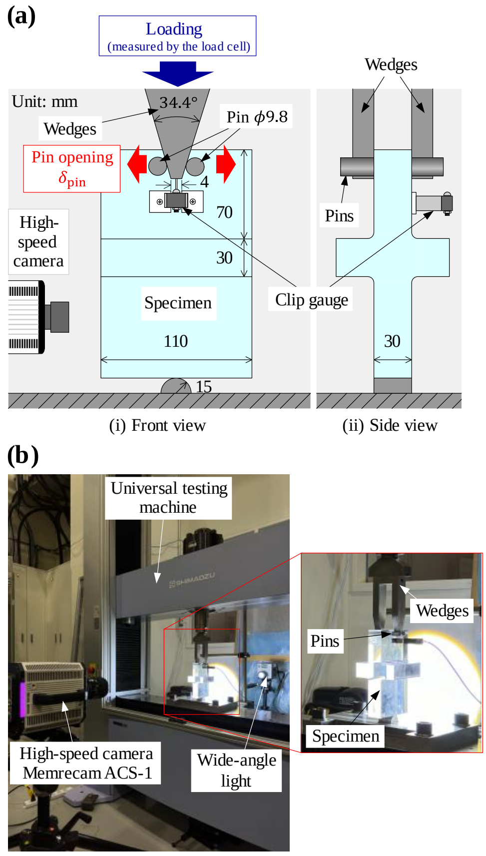
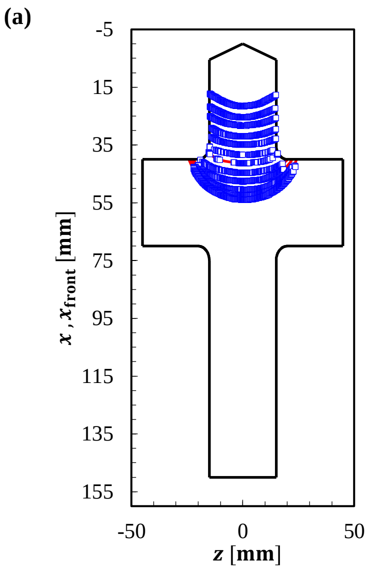


Fig. 3. Experimental procedure of the DCB-type crack arrest test. (a) Schematic of specimen, wedge, pins, and clip gauge, (b) Experimental setup.



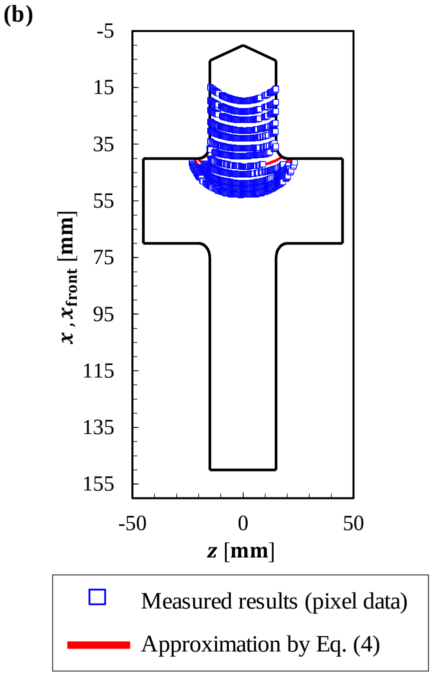


Fig. 4. Crack-shape approximation based on experimental results using Eq. (4). (a) Test J1, (b) Test J2.

The snapshots of the results of the deformed specimens with the σ_{yy} stress fields for test J1 are shown in **Fig. 5**. The proposed strategy robustly simulates the smooth transitions of the local stress fields around the dynamically propagating crack front.

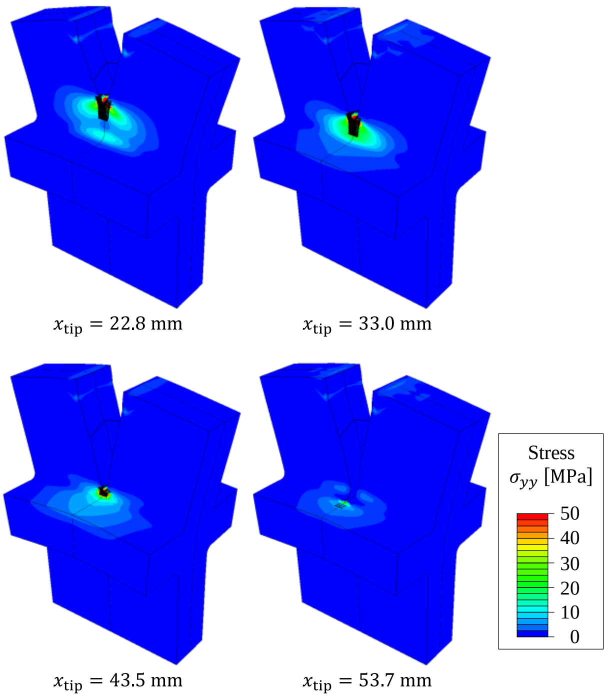


Fig. 5. Results of the generation phase analysis of all deformations and σ_{yy} stress fields at crack arrest for test J1 (scale factor = 20).

Subsequently, we utilised the methodology described in Section 2.3 to calculate the DSIF K_{I} through post-processing. **Fig. 6** shows the resultant relationships between K_{I} and the crack velocity for the respective tests. We observed that both results showed good agreement even though they were independent tests.

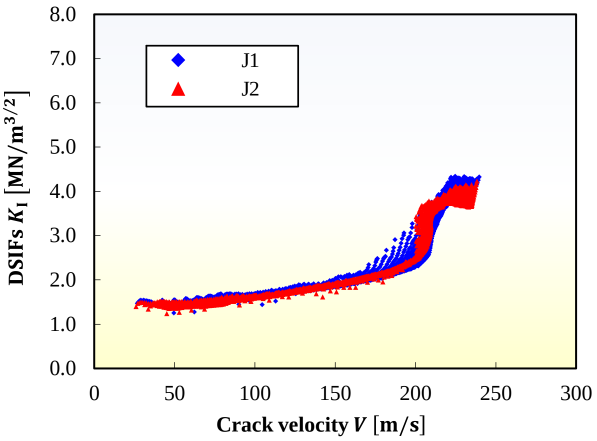


Fig. 6. Relation between DSIFs and crack propagation velocity.

The same methodology describe above has been employed to simulate the high-speed crack propagation in plate structures. And the resultant relationship between DSIFs and crack velocity can be approximated in a unified manner as follows:

$$K_{\mathrm{Id}}(V) = 4.525 \times 10^{-4}\left(\frac{V}{100}\right)^{10} + 0.108\left(\frac{V}{100}\right)^{2} + 1.611, \qquad (6)$$

which was utilised as the material resistance for application phase analysis to predict the crack propagation behaviour in cross-joint structures.

Subsequently, we conducted the application phase analysis, employing the proposed strategy presented in Section 3 to predict the crack propagation behaviours. The prediction results demonstrate that the proposed strategy successfully predicts the history of crack velocity. Notably, the occurrences of crack velocity oscillations, possibly attributed to the vibrations of the specimens during experiments, have been accurately predicted using the proposed strategy. This compelling evidence strongly supports the effectiveness of the proposed strategy.

5. CONSLUSIONS

This study proposes a novel strategy based on the s-method for predicting high-speed crack propagation behaviours in 3D plate structures. The strategy consists of a two-step analysis: a generation phase analysis to establish a material resistance, followed by an application phase analysis utilising this criterion to predict crack propagation behaviours. During the application phase, an optimisation algorithm combining the Non-linear Gradient Descent Method and the Golden Section Method is employed, along with the integration of two criteria for assessing crack arrest. To validate the proposed strategy, we applied it to predict high-speed crack propagation in DCB-type PMMA plate specimens. The generation phase analysis was initially performed using experimental data to establish a material resistance, followed by the application phase analysis to predict crack propagation behaviours. The results successfully replicated tendencies in crack velocity changes, providing strong support for the effectiveness of our strategy.

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