Proceedings of the Conference on Computational Engineering and Science JSCES
Vol. 29, Jun. 2024

Classification of topologically different tetrahedral meshing of rectangular parallelepiped and consideration of its effects in finite element analysis of thin plate

Masaru Muranishieisan[1)], Isao Kobayashi[2)], Masayuki Honchi[2)] and Shigenori Takada[3)]

1) Senior Technologist of Western Digital Technologies GK (Postal Address, E-mail: masaru.muransihi@wdc.com)
2) Retiree of Western Digital Technologies GK
3) Director of Western Digital Technologies GK (Postal Address, E-mail: shigenori.takada@wdc.com)

In an automatic element meshing, various topologically different tetrahedral meshes are obtained. To investigate the effects of the various meshes, the various tetrahedral meshes of the rectangular parallelepiped are classified. The tetrahedral meshing is classified into 9 types. A thin plate was meshed using only 1 type of the 9 types, and the element size dependence of the natural frequency was investigated. In meshes that include slivers, calculation errors become large in modes that include torsional deformation. When meshing without slivers, element dimensions can be increased by about 4 times.

Key Words : *Tetrahedral element, Automatic meshing, Sliver, topologically different mesh*

1. INTRODUCTION

In finite element method (FEM), it is common sense to use fine meshes to obtain highly accurate solutions [1],[2]. In the practical use of FEM, however, the fine meshes gives direct impact to calculation cost in the viewpoint of computing machine power and calculation time. From this point, how to obtain accurate solution from a coarse mesh is a critical issue in practical use of FEM. For example, in thin plate structure, coarse meshes inevitably generate low-quality meshes. This comes from significant difference of the element size in the plate thickness direction and in-plate direction [3]. In some shape of structure, coarse meshes inevitably generate low-quality meshes. How to obtain accurate solution from low-quality meshes is also an important problem in practical use of FEM.

In this paper, the authors take the thin rectangular plate as example, and classify its tetrahedral meshing. It is shown that there exist 9 types of tetrahedral meshing in thin rectangular parallelepiped. The authors compare the modal frequency of thin plate meshed only by one of the 9 types. It is shown that calculation error becomes large in low quality mesh, especially in the combination of the meshing including sliver and the mode including torsional deformation. In addition, it is shown that the meshing algorithm that thin rectangular parallelepiped is divided into 2 triangular prisms and each triangular prism is divided into 3 tetrahedrons is effective to obtain accurate solution from a coarse mesh.

2. CLASSIFICATION OF TETRAHEDRAL MESHING OF THIN RECTANGULAR PARALLELEPIPED

Before categorizing the tetrahedral meshing of the thin rectangular parallelepiped, the tetrahedrons which appear in tetrahedral meshing of the thin rectangular parallelepiped are categorized as shown in Fig. 1. The Z-axis in Fig. 1 shows the direction of plate thickness of thin rectangular parallelepiped. (So, XY plane and the plane parallel to XY plane are front surface and back surface of the thin rectangular parallelepiped. YZ plane, ZX plane, the plane parallel to YZ plane and the plane parallel to ZX plane are side surfaces of the thin rectangular parallelepiped.) The tetrahedrons are categorized by the number of surfaces N_surf which appear at the surface of the thin rectangular parallelepiped. According to “Triangulations” by De Loera et al (2010) [4], the tetrahedrons are named as following. N_surf=3: Corner, N_surf=2: Staircase, N_surf=1: Slanted, N_surf=0: Core.

In Fig. 1, there are 2 Staircases (Staircase-1 and Staircase-2) and 2 Slanteds (Slanted-1 and Slanted-2). In topological viewpoint, Staircase-1 and Staircase-2 are same tetrahedrons (N_surf=2) and Slanted-1 and Slanted-2 are same tetrahedrons (N_surf=1). In Staircase-1 and Staircase-2, however, combination of 2 surfaces which appear at the surfaces of the thin parallelepiped is different. The combinations are as follows.

Staircase-1: not-side surface (front surface or back surface) and side surface

Staircase-2: side surface and side surface

It is natural to distinguish front or back surface from side surfaces in the meshing of thin plate. From this reason, Staircase-1 and Staircase-2 are considered as the different tetrahedrons. In the same way, Slanted-1 and Slanted-2 are considered as the different tetrahedrons.

Slanted-1: side surface

Slanted-2: not-side surface (front surface or back surface)

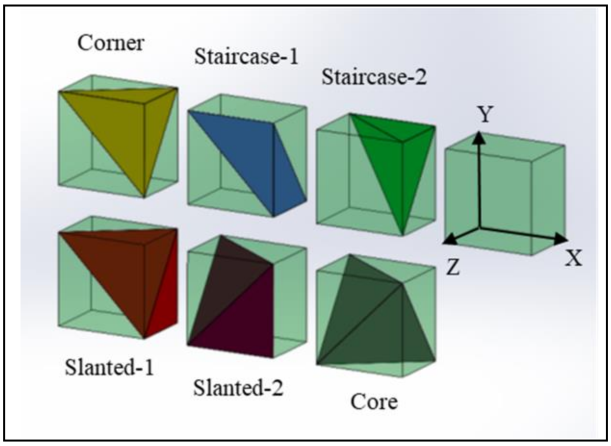

Fig. 1 Categorization of tetrahedrons

Figure 2 is the plane view of the tetrahedrons shown in Fig.1. The shapes of plane view of Slanted-2 and Core are rectangle. The others are triangle. The tetrahedrons like Slanted-2 and Core are called sliver from the shape. The effect of the sliver will be discussed in later.

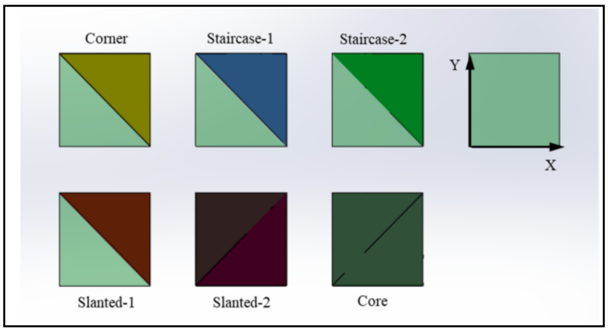

Fig. 2 Plane view of tetrahedron

Using these tetrahedrons, meshing of the rectangular parallelepiped is classified. There exist 9 types of topologically different meshing. The 9 types are shown from Fig. 3 to Fig. 11. Like naming of tetrahedrons, the meshes are named according to "Triangulations" by De Loera et al (2010) [5]. In Fig. 3 to Fig. 11, same colors of tetrahedrons as Fig. 1 are used to distinguish the type of tetrahedron. For example, in Fig. 3, 4 Corners and 1 Core are shown in left hand side. The situation that all tetrahedrons are packed into 1 rectangular is shown in right hand side of Fig. 3. Fig. 3 shows that mesh type-A is constructed by 4 Corners and 1 Core. For example, type-A and type-B are constructed from 5 tetrahedrons and 6 tetrahedrons, respectively. From this point, at least, the fact that type-A and type-B are different meshes can be understood.

In topological viewpoint, Type-C-1 and Type-C-2 are same mesh. These 2 meshes can be converted each other by exchange between front or back surface and side surface (By exchange between front or back surface and side surface, Staircase-1 and Staircase-2 are exchanged, and Slanted-1 and Slanted-2 are exchanged.). The situations are same in Type-D-1 and Type-D-2, and Type-E-1 and Type-E-2. There exist 6 topologically different meshes. In "Triangulations" by De Loera et al (2010) [3], it is written that there exist 6 types of splits of 3-dimensional cube into tetrahedrons. The 2 claims are consistent. As in the case of categorization of tetrahedron, "Type-C-1 and Type-C-2", "Type-D-1 and Type-D-2" and "Type-E-1 and Type-E-2" are considered as the different mesh in thin plate meshing. As will be described later, "Type-C-1 and Type-C-2" have different property in finite element analysis (FEA). "Type-D-1 and Type-D-2" and "Type-E-1 and Type-E-2" are in a similar situation. These distinctions are meaningful in classification of finite element meshing. In this way, tetrahedral meshing of thin rectangular plate is classified as 9 types.

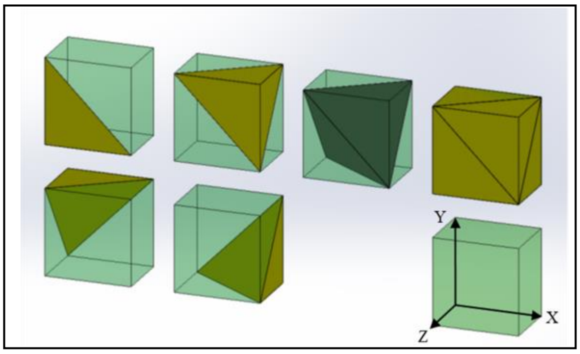

Fig. 3 Tetrahedral mesh Type-A

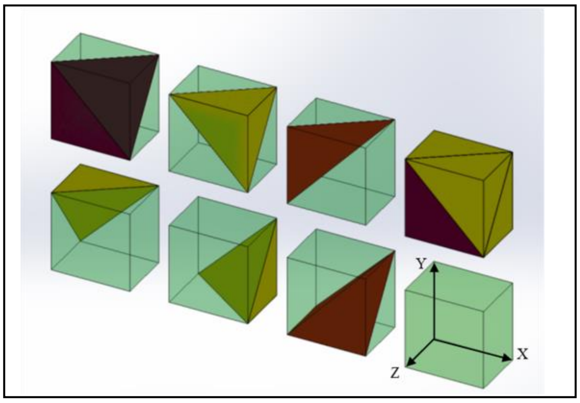

Fig. 4 Tetrahedral mesh Type-B

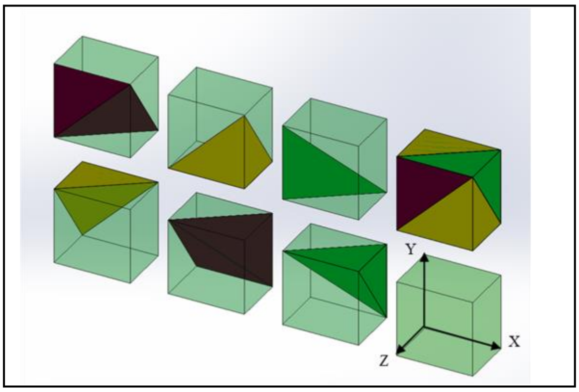

Fig. 5 Tetrahedral mesh Type-C-1

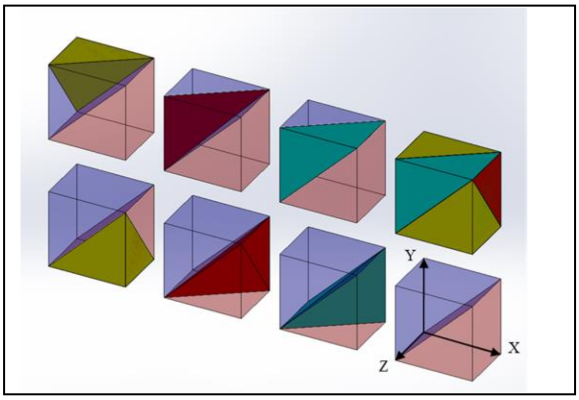

Fig. 6 Tetrahedral mesh Type-C-2

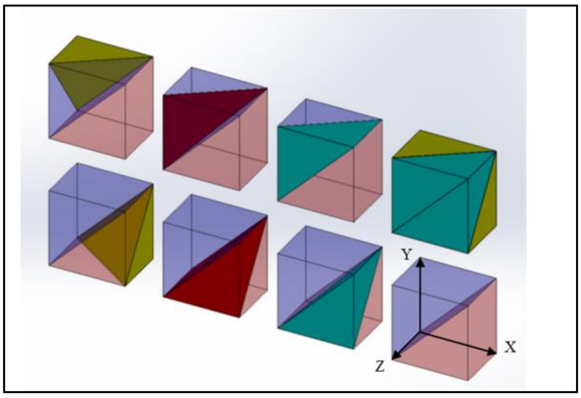

Fig. 7 Tetrahedral mesh Type-D-1

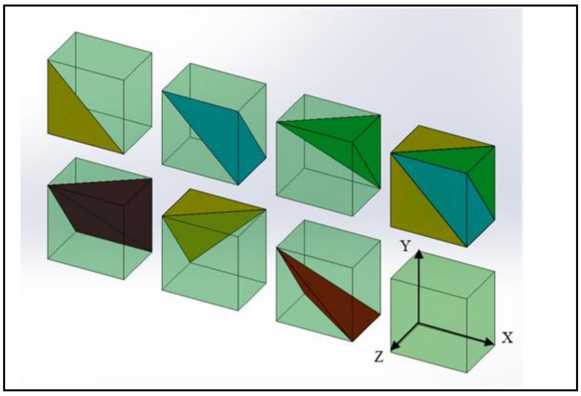

Fig. 8 Tetrahedral mesh Type-D-2

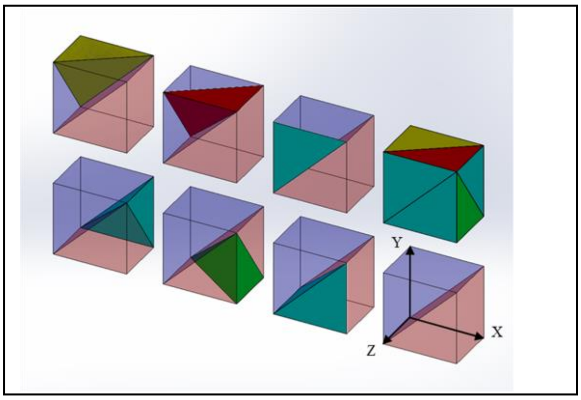

Fig. 9 Tetrahedral mesh Type-E-1

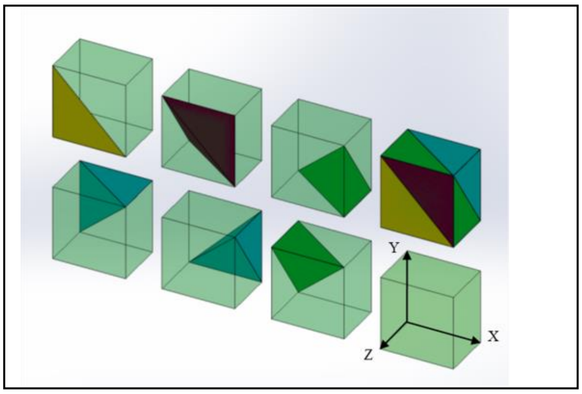

Fig. 10 Tetrahedral mesh Type-E-2

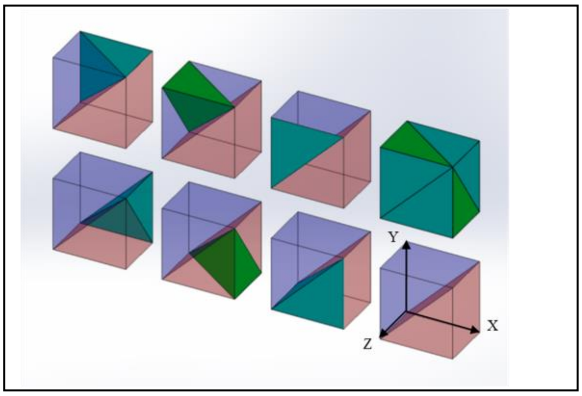

Fig. 11 Tetrahedral mesh Type-F

As shown in Fig. 6, 7, 9 and 11, the thin rectangular parallelepiped is divided into 2 triangular prisms and each triangular prism is divided into 3 tetrahedrons in Type-C-2, Type-D-1, Type-E-1 and Type-F. In contrast, Type-A, Type-B, Type-C-1, Type-D-2 and Type-E-2 are not divided into 2 triangular prisms, because these include sliver (Slanted-2 or Core). The list of the tetrahedrons which construct 9 types of the meshes is shown in Table 1. In the Table 1, the slivers (Slanted-2 and Core) are shaded yellow color.

Table 1 Tetrahedral mesh Type-A

	A	B	C−1	C−2	D−1	D−2	E−1	E−2	F
Corner	4	3	2	2	2	2	1	1	0
Staircase−1	0	0	0	2	2	1	3	2	4
Staircase−2	0	0	2	0	0	1	1	2	2
Slanted−1	0	2	0	2	2	1	1	0	0
Slanted−2	0	1	2	0	0	1	0	1	0
Core	1	0	0	0	0	0	0	0	0

3. EFFECT OF TOPOLOGICALLY DIFFERENT TETRAHEDRAL MESH IN FINITE ELEMENT ANALYSIS OF THIN PLATE

To investigate the effect of topologically different tetrahedral mesh, modal analysis of thin plate is implemented. As shown in Fig. 12, 128mm x 128mm x 1mm thin square plate is meshed into small thin rectangular parallelepipeds. The small thin rectangular parallelepiped is meshed only by one of the 9 types. The material is assumed as steel (Young’s modulus: 200GPa, Poisson’s ratio: 0.3, Density: 7900 kg/m3). In the analysis, tetrahedral quadratic elements are used.

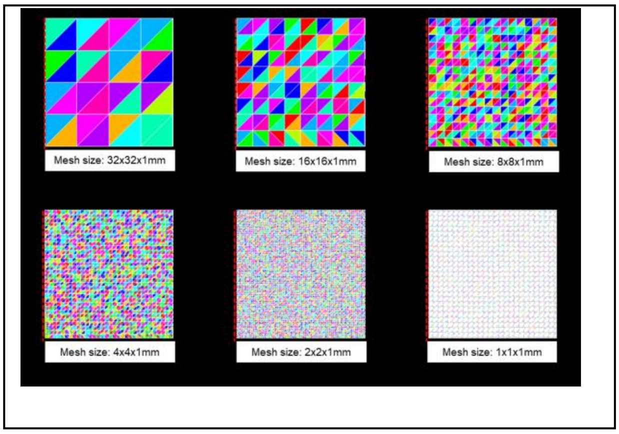

Fig. 12 Mesh of thin plate

The results are shown in Fig. 13 to Fig. 16. For comparison, the results of hexahedral quadratic elements in same mesh size are added (“Hex”). The horizontal axis of the graphs shows ratio of the edge of the small thin rectangular parallelepiped against the plate thickness. For example, the ratio of element size 1 means 1mm x 1mm x 1mm mesh, and the ratio of element size 32 means 32mm x 32mm x 1mm mesh. So, large value of element size ratio means low-quality mesh. The vertical axis shows the ratio of modal frequency against the frequency by the hexahedral quadratic mesh with 1mm x 1mm x 1mm size. This value close to 1 means highly accurate, and large value means low accuracy. At the left-hand side of these figures, the modal shape is shown.

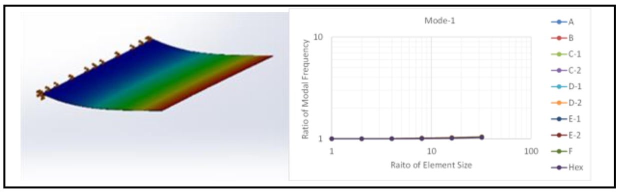

Fig. 13 Mesh size dependency of calculation accuracy (Mode-1).
In all of mesh, property is substantially the same.

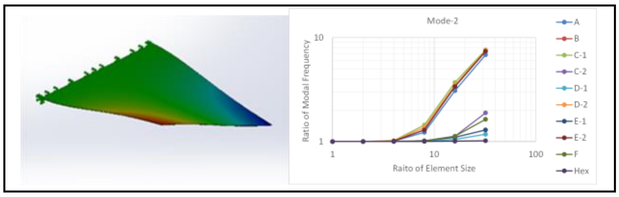

Fig. 13 Mesh size dependency of calculation accuracy (Mode-2).
Especially in Type-A, Type-B, Type-C-1, Type-D-2 and Type-E-2, accuracy is low in low-quality mesh.

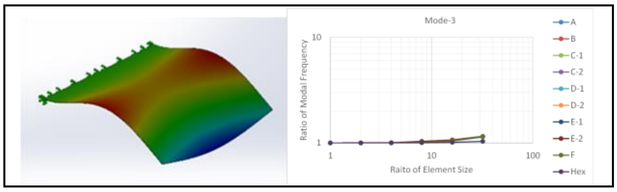

Fig. 14 Mesh size dependency of calculation accuracy (Mode-1).
In all of mesh, property is substantially the same.

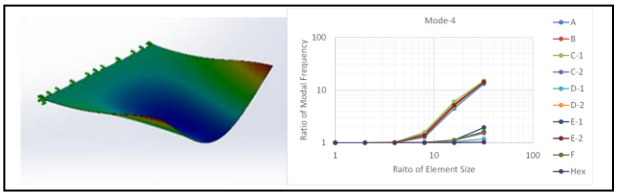

Fig. 16 Mesh size dependency of calculation accuracy (Mode-4).
Especially in Type-A, Type-B, Type-C-1, Type-D-2 and Type-E-2, accuracy is low in low-quality mesh.

In mode-1 and mode-3, element size dependency is small, and difference among mesh types is small. From the mode shapes, these modes are simple bending modes. Compared to these, element size dependency is different by mesh type in mode-2 and mode-4. From the mode shapes, these modes are complex modes that include torsional deformation.

From Table 1, the meshes of Type-A, Type-B, Type-C-1, Type-D-2 and Type-E-2 include sliver. It is estimated that the origin of element size dependency in Type-A, Type-B, Type-C-1, Type-D-2 and Type-E-2 is the slivers, and the mesh size can be extended about 4 times by the mesh without sliver.

4. ORIGIN OF THE DIFFERENCE OF VIBRATION PROPERTIES

As mentioned above, element size dependency of modal frequency is different in topologically different tetrahedral meshing. As the origin of the difference of the vibration properties, mass and stiffness can be assumed as the candidates that characterize modal frequency. The authors thought that the stiffness affects the modal frequency and implemented static deformation analysis shown in Fig. 17 is implemented. In this static analysis, the same finite element models as the modal analysis are used.

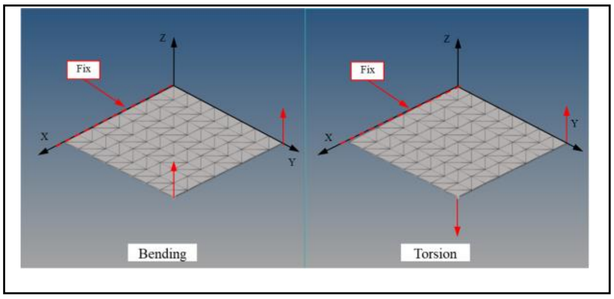

Fig. 17 Boundary condition of static analysis

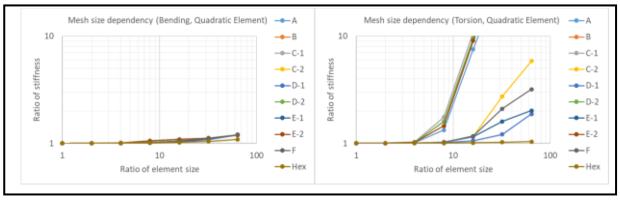

Fig. 18 Boundary condition of static analysis

The results are shown in Fig. 18. The vertical axes are shown the ratio of stiffness compared to the values of the hexahedral quadratic mesh in the mesh size of 1mm x 1mm x 1mm. The ratio of torsional stiffness suddenly increases at the ratio of element size more than 8 in Type-A, Type-B, Type-C-1, Type-D-2 and Type-E-2. In bending stiffness, the mesh type dependency is small. This element size dependency is consistent with the element size dependency of modal frequency. From this, it is thought that the element size dependency of modal frequency comes from the element size dependency stiffness.

In the next step, the origin of high stiffness in low quality mesh in Type-A, Type-B, Type-C-1, Type-D-2 and Type-E-2 which have slivers is considered. The deformation shapes of above static analysis results are shown in Fig. 19. In the bending condition, the deformation shape is curvilinear in Y direction and roughly constant in X direction. To simulate this shape, a interpolation function of the form Y^2 is needed in the elements. In the torsional condition, the deformation shape is curvilinear in Y direction and linear in X direction. To simulate this shape, a interpolation function of the form $X \times Y^2$ is needed in the elements. The interpolation function of a tetrahedral quadratic element has only up to a quadratic function of X, Y and Z. So, the interpolation function of a tetrahedral quadratic element does not have the function of the form $X \times Y^2$. From this, it is thought that tetrahedral quadratic elements are basically incapable of representing torsional deformation of thin plate as shown in Fig.19. (The interpolation function of a tetrahedral quadratic element has the function of the form $aX^2 + bY^2 + cXY$. Presenting simple bending deformation in one direction of thin plate is capable.)

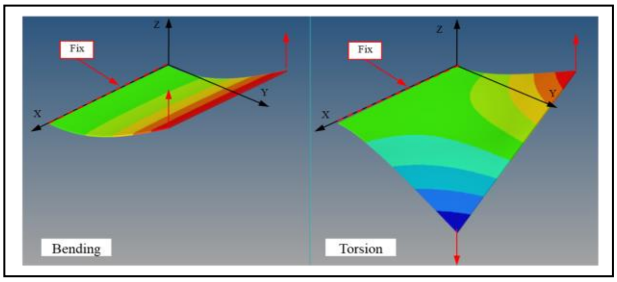

Fig. 19 Deformation shape of static analysis results

However, if the thin rectangular parallelepiped is divided into 2 triangular prisms like Type-C-2, Type-D-1, Type-E-1 and Type-F, the deformation as shown in Fig. 20 is possible (This deformation shape is just one of examples.). This kind of deformation can simulate torsional deformation. Even if the deformation of each triangular prism cannot simulate torsional deformation, the combination of deformations of 2 triangular prisms can simulate torsional deformation. The authors estimate that the origin of high stiffness in low quality mesh in Type-A, Type-B, Type-C-1, Type-D-2 and Type-E-2 (which have slivers) is a kind of locking phenomenon caused by the inability of interpolation functions to represent deformations [6].

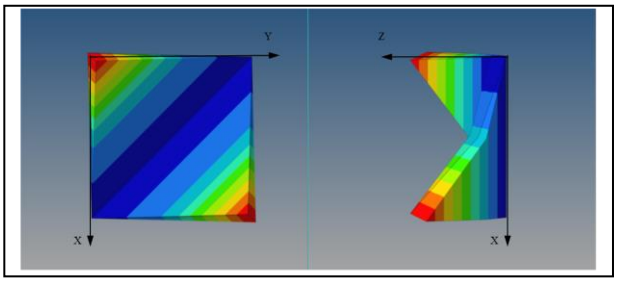

Fig. 20 Deformation shape of static analysis results

To confirm this estimation, static analysis shown in Fig.17 is implemented using tetrahedral cubic elements [7]. The results are shown in Fig. 21. The high stiffness in low quality mesh shown in the right-hand side of Fig. 18 is disappeared not only in Type-A, Type-B, Type-C-1, Type-D-2 and Type-E-2, but also in Type-C-2, Type-D-1, Type-E-1 and Type-F. The interpolation function of tetrahedral cubic elements is constructed by cubic function of X, Y and Z, and has the function of the form $X \times Y^2$.

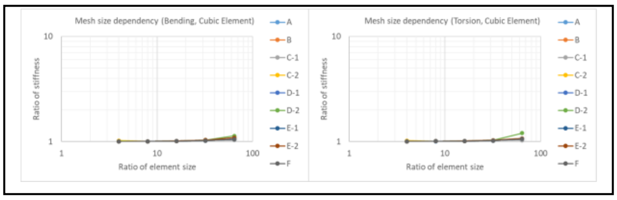

Fig. 21 Results of static analysis using cubic elements

Even in high quality mesh (the ratio of element size is relatively small), the meshes which have slivers cannot simulate torsional deformation (The interpolation function does not have the function of the form $X \times Y^2$.). In the situations that the small thin rectangular parallelepiped meshes are small enough, however, the deformation in such small rectangular parallelepiped in the torsional deformation of thin plate as shown in Fig. 18 can be approximately regarded as simple bending. It is thought that this is the reason that all of 9 types gives accurate solution in high quality mesh.

5. CONCLUSION

In tetrahedral meshing of thin rectangular plate, the effect of topologically different meshing is investigated. From the investigation, the following conclusions are obtained.

(1) In the tetrahedral meshing of thin rectangular parallelepiped, there exist 9 types of topologically different meshing. In all of 9 types, if mesh size is roughly less than 4 times of plate thickness, calculation accuracy substantially same as hexahedral quadratic elements can be obtained. The authors think that this is matched with the common sense to use fine meshes to obtain highly accurate solutions.

(2) The 9 types are classified into 2 classes. In the 1st class (Type-C-2, Type-D-1, Type-E-1 and Type-F), the thin rectangular parallelepiped is divided into 2 triangular prisms and each triangular prism is divided into 3 tetrahedrons. In the 2nd class (Type-A, Type-B, Type-C-1, Type-D-2 and Type-E-2), the thin rectangular parallelepiped is not divided into 2 triangular prisms because of the slivers. The mesh size of thin rectangular plate can be expanded about 4 times by use of the 1st class (Type-C-2, Type-D-1, Type-E-1 and Type-F). The meshing algorithm corresponds to the 1st class is effective to obtain accurate solution from a coarse mesh.

The followings are future work.

(1) It is thought that tetrahedral cubic element is effective to obtain accurate solution from a low-quality mesh. However, node number of one tetrahedral cubic element (20) is more than node number of one tetrahedral quadratic element (10). Use of tetrahedral cubic element creates new tradeoff between cost and accuracy. This tradeoff will be discussed in the following paper.

(2) For the mesh quality, various parameters like aspect ratio, skew, etc. are defined. However, the relationship between mesh quality parameter and topologically different meshing is not discussed. The relationship with mesh quality parameters will be discussed in the following paper.

.

REFERENCES

[1] Dutt A., Effect of Mesh Size on Finite Element Analysis of Beam, SSRG International Journal of Mechanical Engineering, Vol.2, Issue 12 (2015), pp.8–10.

[2] More S. T., Effect of Mesh Size on Finite Element Analysis of Plate Structure, International Journal of Engineering Science and Innovative Technology, Vol.4, Issue 3 (2015), pp.181–185

[3] Zienkiewicz O. C, R. L. Taylor and J. Z. Zhu, The Finte Element Method: Its Basics & Fundamentals, 7th edition (2013), p407, Butterworth and Heinemann

[4] De Loera J. A., Rambau J. and Santos F., Triangulations (2010), p319, Springer

[5] De Loera J. A., Rambau J. and Santos F., Triangulations (2010), p21, Springer

[6] Yanada T, Kuseinou Yugen Youso Hou (2007), p56, Maruzen (in Japanese)

[7] Zienkiewicz O. C, R. L. Taylor and J. Z. Zhu, The Finte Element Method: Its Basics & Fundamentals, 7th edition (2013), p163, Butterworth and Heinemann