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Indispensability of Subloading Surface Model with Overstress Model for Descriptions of Irreversible Mechanical Phenomena of Solids
-*Governing Law of Irreversible Mechanical Phenomena of Solids*-

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The elastoplastic deformation including the general cyclic loading behavior can be pertinently described only by the subloading surface model. Further, the elastoplastic deformations of materials other than metals and the other irreversible deformations, e.g. the damage phenomenon and the sliding (friction) phenomenon can be also described only by the subloading surface model. The finite deformation can be described by the subloading-multiplicative hyperelastic-based plastic/viscoplastic model for not only the monotonic but also cyclic loading process. Further, the elastoplastic and the elasto-viscoplastic models can be unified to the subloading-overstress model describing the irreversible deformation at the general rate, and the elastoplastic and the elasto-viscoplastic friction (sliding) models can be unified to the subloading-overstress friction (sliding) model for the general sliding rate. Consequently, the elastoplastic deformation model and the elastoplastic friction (sliding) model can be disused. Consequently, t*he subloading surface model can be regarded as the governing law of the irreversible mechanical phenomena of solids*. Nowadays, we encounters with the epoch-making development of the solid mechanics by the creation of the subloading surface model.

Key Words : *Subloading surface model, Irreversible deformation, Rate-dependent behavior, Multiplicative finite strain theory, Overstress model, Cyclic loading, Damage phenomena, Friction phenomena.*

1. INTRODUCTION

Various plasticity model models, e.g. the two surface model (Dafalias and Popov [1][2]), the Chaboche model [3] (see also Ohno and Wang [4]) and the subloading surface model (Hashiguchi [5]-[8]) are installed in various commercial software. Among them, only the subloading surface model is capable of describing the cyclic loading behavior as reviewed in detail by Hashiguchi [8] and Hashiguchi et al. [9][10]. Further, these elastoplastic models except for the subloading surface model are concerned only with the elastoplastic deformation of metals. Nevertheless, they are incapable of describing the plastic strain rate in the unloading process, so that the general cyclic loading behavior including the pulsating loading cannot be described by them. Furthermore, the elastoplastic deformations of materials other than metals, e.g. soils, glass, rocks, concretes and the other mechanical phenomena, e.g. the damage phenomena including the unilateral and the ductile damages and the friction phenomenon can be described only by the subloading-unilateral damage model [11], the subloading-Gurson model and the subloading friction model (Hashiguchi et al. [13], Hashiguchi and Ozaki [14], Hashiguchi and Ueno [15]).

Further, the general formulation for the finite deformation in the multiplicative hyperelastic-based viscoplasticity even for the cyclic loading process has been attained by the subloading-multiplicative elastoplastic model [16], although it has been limited to the conventional elastoplastic constitutive equation with the yield surface enclosing the purely-elastic domain.

Moreover, the elastoplastic and the elasto-viscoplastic models can be unified to the subloading-overstress model describing the irreversible deformation at the general rate, and the elastoplastic and the elasto-viscoplastic friction (sliding) models can be unified to the subloading-overstress friction (sliding) model for the general sliding rate. Consequently, the elastoplastic deformation model and the elastoplastic friction (sliding) model can be disused as described in detail by Hashiguchi [8] and Hashiguchi et al. [9][10][17].

2. FUNDAMENTAL CONCEPT OF SUB-LOADING SURFACE MODEL

The plastic strain rate is induced by the mutual slips between material particles. Here, the mutual slips are not induced simultaneously but they are induced gradually as the stress approaches the yield surface. Then, it is assumed in the subloading surface model that *the plastic strain rate is induced as the stress approaches the yield surface or inversely the stress approaches the yield surface as the plastic strain rate is induced, exhibiting a continuous variation of tangent stiffness modulus, but the stress recedes from the yield surface when only the elastic strain rate is induced.*

In this context, it is first required to incorporate the general measure which describes the approaching degree of stress to the yield surface, renamed the *yield surface*, in order to formulate the plastic strain rate. To this end, we introduce the following ***subloading surface*** which always passes through the current stress and maintains a similar shape and a same orientation to the normal-yield surface.

3. A LOT OF MECHANICAL ADVANTAGES OF SUBLOADING SURFACE MODEL FOR DESCRIPTIONS OF IRREVERSIBLE MECHANICAL PHENOMENA

The subloading surface model possesses a lot of advantages for describing the plastic/viscoplastic deformation, the damage phenomenon and the friction phenomenon, etc. as will be delineated in the following.

3.1 Smooth elastic-plastic transition

The smoothness condition (Hashiguchi [18][19]) is satisfied only in the subloading surface model and thus smooth transition from the elastic deformation to the elastoplastic state observed in test data of real materials can be described only by this model. On the other hand, This condition is violated and thus the abrupt transition from the elastic to the elastoplastic state is described in all the other models.

3.2 Continuous variation of tangent stiffness modulus

The continuous variation of the tangent stiffness modulus is always described by the subloading surface model but the sudden decrease of the tangent stiffness modulus is described at the moment when the stress reaches the yield surface in the all other models.

3.3. Simplicity in loading criterion: Unnecessity of yield judgment

The judgement whether the stress reaches the yield surface in the loading criterion for the plastic strain rate is not required in the subloading surface model (Hashiguchi [20][21]) but it is required in all the other models.

3.4 Description of plastic strain rate in unloading process: Rational description of *spring-back phenomenon*

The plastic strain rate in the unloading process is described in the subloading surface model but it cannot be described in the two surface model (Dafalias and Popov [1][2]), the Chaboche model [3] (see also Ohno and Wang [4]). Therefore, the spring-back phenomenon is described appropriately by the subloading surface model but it cannot be described pertinently by the other models predicting the unrealistically small springback caused only by the elastic deformation. Nevertheless, Yoshida’s group (e.g. Yoshida and Uemori [22] Yoshida and Amaishi [23]) insists that the Yoshida-Uemori model is capable of describing the springback phenomenon. Here, it should be noticed that Yoshida-Uemori model is based on the two surface model which is incapable of describing the plastic deformation in the unloading process and then incorporates the *chord modulus* connecting the initial point of the unloading and the unloaded point to the zero stress and let it be decrease with the equivalent plastic strain but let the decreasing rate cease by the exponential function of the equivalent plastic strain. Here, it should be noticed that the chord modulus is physically irrational idea against the foundation of the continuum damage mechanics insisting that the damage proceeds acceleratingly once it is induced in the monotonic loading process. Consequently, it should be noticed that the application of this model to the mechanical design of mechanical elements subjected to the cyclic loading causes the dangerous accidents, since the plastic deformation cannot be described during the pulsating loading process.

It should be noticed that only the subloading surface model is capable of describing the plastic strain rate in the unloading process and thus the springback phenomenon is described rigorously only by the subloading surface model as described by Hashiguchi [8], Hashiguchi et al. [10].

3.5 Closed hysteresis loop in pulsating loading process

The closed hysteresis loop in the pulsating loading process, i.e. the positive or the negative one side stress cyclic loading process is described by the subloading surface model but it cannot be described by the two surface model [1][3] and the Chaboche model [3] (also Ohno-Wang model [4]) which describe the open hysteresis loop leading to the excessively large mechanical ratchetting. It would be immoral as scientists that they have never shown the responses of their models for the pulsating loading.

3.6 Description of general cyclic loading behavior

The general cyclic loading behavior can be described rigorously only by the subloading surface model but it cannot be described realistically by the other models (Hashiguchi [8][9][10]). The mechanical design of machineries subjected to the vibration by use of the elastoplastic models, e.g. Chaboche model, Ohno-wang model and Yoshida-Uemori model other than the subloading surface model induces seriously-dangerous accidents.

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3.7 Automatic controlling function to pull back stress to yield surface

The subloading surface model contains the automatic controlling function to pull-back the stress to the yield surface, so that quite large strain increments can be input in a numerical calculation, realizing a high efficiency (Hashiguchi [8]). However, it is required for the elastoplastic models other than the subloading surface model to input quite small strain increments such that the stress does not go out from the yield surface in the plastic loading process.

3.8 Description of finite deformation in hyperelastic-based plasticity

Formulation for the finite deformation in Multiplicative hyperelastic-based plasticity has been limited to the conventional elastoplastic constitutive equation with the yield surface enclosing the purely-elastic domain.

The multiplicative hyperelastic-based plasticity has been studied centrally by Simo and his colleagues (e.g. Simo [32], [33], [34], [35], Simo and Pister [36], Simo and Ortiz [37], Simo and Taylor [38], Ortiz and Simo [39]) in the last century, in which the logarithmic strain has been used mainly leading to the co-axiality of stress and strain rate so that it has been limited to the isotropy. It has been developed actively from this century on by Lion [40], Menzel and Steinmann [41], Wallin et al. [42], Dettmer and Reese [43], Menzel et al. [44], Wallin and Ristinmaa [45][46], Gurtin and Anand [46], Sansour et al. [47], Vladimirov et al. [48][49], Henann and Anand [50], Brepols et al. [51], etc. in which constitutive relations are formulated in the intermediate configuration imagined fictitiously by the hyperelastic unloading to the stress-free state. However, the plastic flow rule with the generality for the elastically anisotropy remains unsolved and only the *conventional plasticity model* with the yield surface enclosing the elastic domain have been incorporated so that only the monotonic loading behavior of elastically-isotropic materials are concerned in them.

The general finite strain theory within the framework of the multiplicative hyperelastic-based plasticity not only for the monotonic loading but also for the cyclic loading process is formulated rigorously by the subloading surface model (Hashiguchi [16][8][52]).

3.9 Deformation behavior in general rate: Unification of elastoplastic and viscoplastic constitutive equations

The constitutive models for the description of the viscoplastic deformation behavior are divided into the creep model and the overstress model. The former possesses the physically irrational mechanical structure that it is not reduced to the elastoplastic equation for the quasi-static rate of deformation, while the overstress model is reduced to the elastoplastic equation for the quasi-static rate of deformation. Unfortunately, however, the creep model is adopted widely (e.g. Lemaitre and Chaboche [24], Lubarda [25], de Souza Neto et al. [26], Ohno et al. [27]).

The subloading-overstress model is capable of describing the viscoplastic deformation behavior from the quasi-static to the impact loading behavior (Hashiguchi [8], Hashiguchi et al. [9][10]). Consequently, *the elastoplastic constitutive equation can be disused by using only the subloading-overstress model*.

The plastic strain rate is described by the stress rate in the plastic constitutive equation. On the other hand, the viscoplastic strain rate is described in terms of the state variables, so that the numerical calculation is simplified drastically. Therefore, the constitutive relation is described by the drastically simplified equation.

3.10 Unilateral damage for brittle materials and generalized Gurson model extended to describe cyclic loading for ductile materials

It is quite difficult to describe the constitutive equation of the unilateral damage phenomenon by the stress rate vs. strain rate relation in the principal stress directions. Then, the return-mapping calculation is adopted by de Souza Neto et al. [26]. On the other hand, it can be described concisely by the subloading-overstress model as formulated by Hashiguchi [11], since the viscoplastic strain rate is described in terms of the current variables without using any rate variable.

Further, Gurson model for the ductile damage of metals can be formulated by the subloading-Gurson model [12] so as to be able to describe the cyclic loading behavior.

3.11 Applicability to wide classes of materials and phenomena

Most of the elastoplastic models other than the subloading surface model are concerned only to metals, assuming the Mises yield surface. On the other hand, the subloading surface model is concerned to the wide classes of materials including metals, soils, glass, concrete, rocks, polymers, etc. (Hashiguchi et al. [8][28][29][30]) with the unilateral damage phenomenon (Hashiguchi [11]) described in the former section. The friction phenomenon is also described only by the subloading surface model as will be described in the next section.

Besides, the crystal plasticity and Mullins effect in polymers, etc. are also formulated rigorously by the use of the subloading surface model (Hashiguchi [8]).

3.12 Precise description of fiction phenomenon

The elastoplastic models other than the subloading surface model are incapable of describing the friction phenomenon. The friction phenomenon has been described by Coulomb friction law and unfortunately it is installed in almost all the commercial software, in which the plastic tangential displacement proceeds at the constant tangential contact stress suddenly after the tangent contact stress reaches the Coulomb friction condition in the monotonic sliding process. Further, the tangent contact stress increases proportionally with the increase of the normal

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contact stress. Therefore, it leads to the unrealistic phenomenon that the tangent contact stress exceeds the limit of the shear strength of the contact bodies.

The friction phenomenon is expressed rigorously by the subloading friction model (Hashiguchi et al. [13], Hashiguchi and Ozaki [14], Hashiguchi and Ueno [15]) which is capable of describing 1) the reduction from the static friction to the kinetic friction 2) the recovery to the static friction during the stop of sliding and further 3) the decrease of the ratio of the tangential contact stress to the normal contact stress with the increase of the normal contact stress (Hashiguchi and Ueno [15]). Therefore, the subloading friction model is indispensable to predict the loosening of bolt and nut fastening, the occurrence of an earthquake. Further, not only the dry friction but also the fluid friction can be described in the unified equation by the subloading-overstress friction model [8][17].

Without adopting the subloading friction model, the loosening of bolts nuts, the occurrence of earthquake, etc. can never be predicted forever.

4. CONCLUSIONS

Chaboche model [3] is installed in a lot of commercial FEM software, i.e. Abaqus, Marc, ANSYS, Ls-dyna, etc., Ohno-Wang model [4] is installed in the commercial FEM software, i.e. FINAS/STAR (CTC, Ltd.), Adventure cluster (SCSK, Ltd.),etc. and Yoshida-Uemori model [22][23] is installed in the commercial software, i.e. Ls-dyna, Pam-Stamp， AutoForm, JSOL-Jstamp, etc. for the springback analysis of thin plate forming. However, these models are incapable of describing the plastic strain rate in the unloading process as far as the physically irrational method seen in Yoshida-Uemori model [22][23] is not used, so that the mechanical designs by these models would result in the dangerous accidents. Besides, these models do not possess the generality as they are relevant only to the elastoplastic deformation behavior in the monotonic loading of Mises metals but irrelevant to the other deformation behaviors of materials other than Mises metals and to the friction phenomenon. Therefore, the installations of these models to commercial software will have to be stopped hereinafter for the sound mechanical designs of industrial products. On the other hand, the subloading surface model is installed in the commercial software Marc (MSC Software, ltd.), Adventure cluster (SCSK, Ltd.) and COMSOL Multiphysics, etc. at present but will have to be installed widely hereinafter.

It can be concluded that the subloading becsurface model is indispensable for the description of the irreversible mechanical phenomenon of solids. Therefore, it can be concluded that

The subloading surface model is the governing law of the irreversible mechanical phenomena of solids.

Besides, the formulations of constitutive equations would become increasingly sophisticated with the development of constitutive equations. On the other hand, it is desirable that elaborate constitutive model is widely adopted for the mechanical design in industries. To this aim, it is desirable to determine the material parameters easily even if details of constitutive formulation are not understood. The article by Liu er al. [53] would contribute to this aim, in which the automatic determination procedure of the material parameters in the subloading surface model by use of the artificial neural networks is shown. The further development of research from these aspects is desirable in particular.

Nowadays, we are encountering with the epoch-making historical development of the solid mechanics by the creation of the subloading surface model.

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