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Data-driven Topology Design with Fragmentation Learning for High-Dimensional Problems

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This study presents a method for tackling high-dimensional topology optimization problems. Our method is based on data-driven topology design, which is a gradient-free topology optimization method using a deep generative model. As a key idea, we propose a block-based fragmentation learning that fragments high-resolution image data into smaller blocks for realizing the efficient learning process. We demonstrate the potential of the proposed method through an L-shaped beam topology optimization example.

Key Words : *Data-driven topology design, fragmentation learning, image-to-image translation*

1. Introduction

Topology optimization allows any shape to emerge within design constraints, resulting in a high degree of design freedom, by replacing the structural optimization problem with a material distribution problem in the design domain [1]. However, dealing with highly nonlinear physical phenomena, such as turbulence, the multi-modal nature of the solution space makes it easy to trap bad local optima using the gradient method. Thus, data-driven topology design provides a gradient-free way to solve topology optimization problems in various physical fields [2,3].

The core of the data-driven topology design framework, as demonstrated by the seminal works cited [2,3,4,5], lies in leveraging a deep generative model, specifically a variational autoencoder (VAE), to exhaustively generate various candidate solutions. And Nie et al. [6] have successfully applied conditional generative adversarial networks (cGANs), utilizing inputs from diverse physical fields within the unaltered material domain, which has led to notable reductions in error rates when addressing problems featuring previously unseen boundary conditions.

Since the deep generative model must be trained at each optimization step in data-driven topology design, the training cost should be relatively low. As the number of input design variables increases, the computational cost of the VAE will be higher when the number of layers and the latent space are larger, and it causes a limitation to about 10^4 variables. Consequently, applying data-driven topology design becomes particularly challenging for high-dimensional problems, such as those involving high-resolution and three-dimensional issues, where the dimensionality of the design variables is much more enormous.

In response, we propose a new framework that can be applied to high-dimensional problems by incorporating the image transformation neural network based on pix2pix [7] that leans a fragment of the structure into data-driven topology design. In this framework, high-dimensional raw data are converted into low-dimensional data, which are considered as inputs to the VAE that have been downscaled so that the structure of the VAE does not have to be too complex to compute. The output of VAE of low-dimensional is converted to the high-dimensional material distribution by the neural network to obtain new high-dimensional solution candidates. The proposed method converts the entire structure between low and high dimensions by tiling neural network. The expectability of the proposed method is discussed through applying it to an L-shaped beam problem.

2. Data-driven topology design

(1) Main concept

The original work by Yamasaki et al. [2] introduces a data-driven topology design approach designed to optimize material distribution within a predetermined design domain, starting from initial material configurations to achieve enhanced performance outcomes. Training data are mapped into a latent space following a specific probability distribution, enabling the generation of new material distributions that inherit features from the training data through sampling within this latent space. These newly generated distributions are then incorporated into

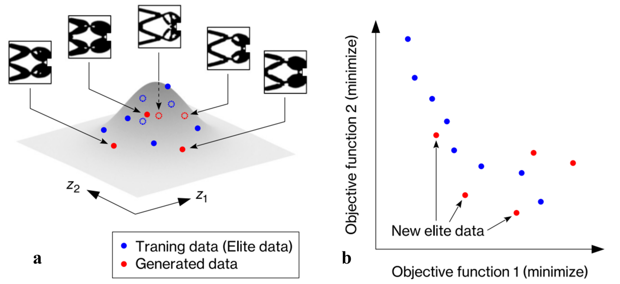


Fig. 1: Basic idea of selecting elite data in data-driven topology design [2]

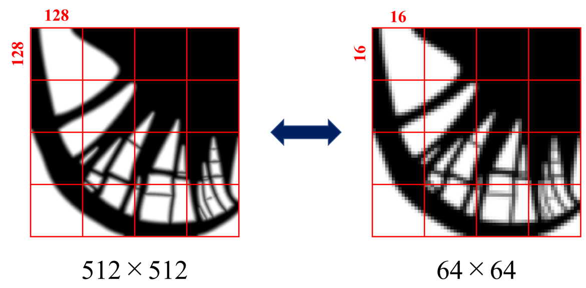


Fig. 2: Pasting fragmentation process to convert between high-dimensions and low-dimensions

the dataset. Through iterative selection of superior material distributions for training data updates, the performance of these distributions is enhanced using a data-driven method. This iterative, performance-focused selection process is depicted in **Fig.1**, illustrating the methodology for identifying and integrating elite material distributions into the training iterations.

(2) Limitation of large-scale problems

Data-driven topology design deals with variables up to 10^4 dimensions. However, this approach relies on VAE with a small architecture, limiting its success when dealing with larger dimensions. To address this limitation, the number of nodes and intermediate layers of VAE should increase, leading to larger architectures. Nevertheless, this

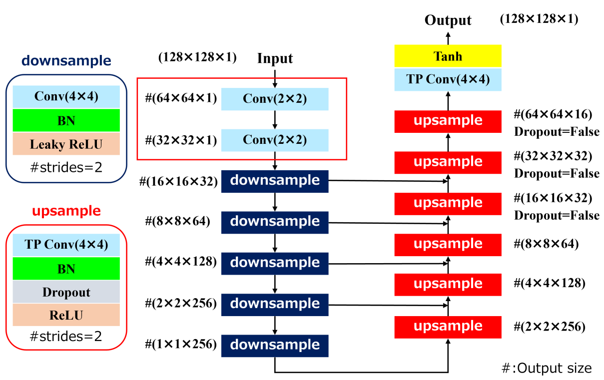


Fig. 3: Architecture of generator

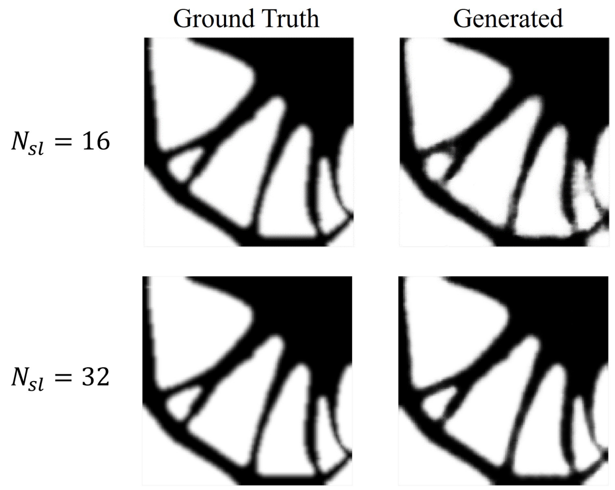


Fig. 4: Comparison of different type of generator with the test data

solution comes with several challenges.

Increasing the nodes and intermediate layers leads to a rise in computational costs per iteration, impacting the overall computational cost. Moreover, accommodating larger network architectures necessitates an increase in the dimensions of the latent space. However, in data-driven approaches, solutions are often searched for through limited random sampling within the latent space. An overly large latent space complicates efficient sampling, thus affecting the efficiency of solution searching. These difficulties makes it challenging to apply data-driven topology design to large-scale problems.

3. Fragmentation learning

(1) Basic concept

In this section, we introduce a neural network-based proxy representation, called ”fragmentation learning”, which incorporates important dimensionality reduction as well as dimensionality enhancement techniques, which can be used to deal with structures of high computational complexity. This approach works by segmenting a image into numerous square, small-sized elements, and then performing dimensionality reduction and upscaling on each small-sized fragment separately.

Specifically, for the original data of size 512×512, we segment it into smaller square elements of size 128×128, as illustrated in **Fig.2**. The neural network then processes and learns from these elements sequentially, enhancing the efficiency of handling complex parsing structures. From left to right, we need to downscale the high-dimensional data to low-dimensional data, which will be used as input to the VAE. From right to left, we need to regenerate the high-dimensional data from the new VAE generating data. Instead of generating the image as a whole, the image can be generated block by block with an image transform neural network as follows, named pix2pix [7]. This technique not only simplifies the processing of intricate parsing structures, but also greatly improves the computational ef-

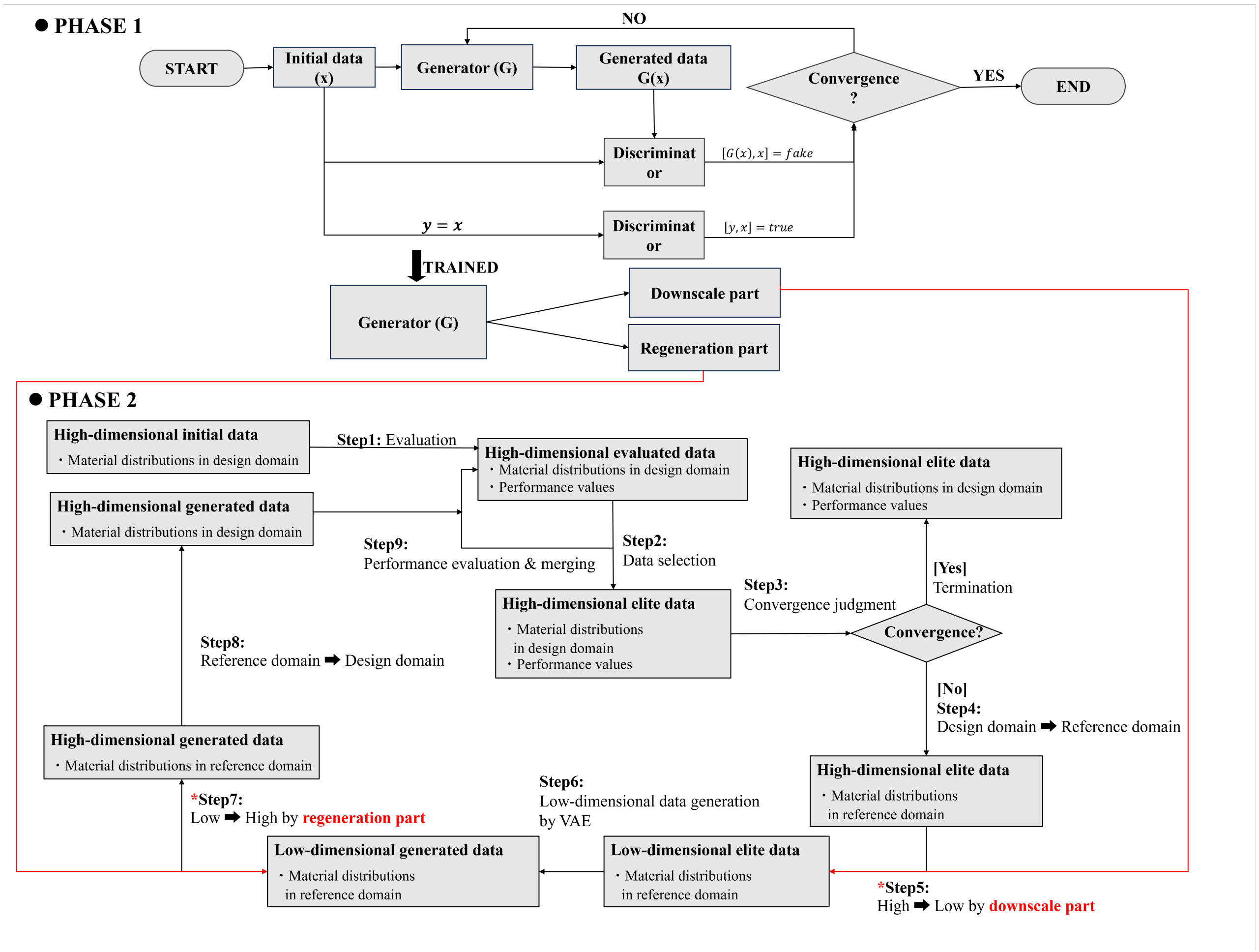


Fig. 5: Overall proposed framework. **phase 1:** training process of pix2pix; **phase 2:** data-driven topology design combined with fragmentation learning

ficiency of the process.

(2) Pix2pix from image transformation

In this study, we utilize a neural network known as pix2pix for image transformation, as detailed in the work of Isola et al. [7]. This network employs a conditional generative adversarial network (cGAN) framework, which includes a generator, G, that maps an input image x, and a noise vector z to an output image y, following the function $G : [x, z] \to y$. The objective of the generator G is to create images indistinguishable by the discriminator D, as either real or fabricated. In contrast, the role of a discriminator is to accurately classify the pair of the real output image y, and the input image x, represented as $[y, x]$, as a real image, and the combination of the generated image, $G(x)$, with the input image, x, denoted as $[G(x), x]$, as a fake image. The dynamic interactions between the generator and the discriminator in the cGAN framework illustrate the adversarial process that drives the network to generate highly realistic images, making both the generator and the discriminator more powerful during training.

Our primary objective is to develop methodologies for both downscaling and upscaling images. To achieve this, we employ a pix2pix model, initially trained with identical inputs and outputs (400 topology optimization result maps under a wide variety of working conditions and boundary conditions as training objects). Subsequently, this model is bifurcated into two distinct components: one for downscaling (high→low), named "downscale part"; the other for upscaling (low→high) can be called "regeneration part". The generator architecture utilized in our research is illustrated in **Fig.3**, featuring a high-dimensional part with a size of $N_{sh} = 128$ and a low-dimensional part with a size of $N_{sl} = 32$. Throughout the training phase, the convolutional layers highlighted in the red box in **Fig. 3** have their weights fixed at 0.25, and no further training is conducted on these layers. This setup enables one network transforms from high-dimensional ($N_{sh} \times N_{sh}$) to low-dimensional ($N_{sl} \times N_{sl}$), and vice versa for the second network. The value of N_{sl} is an essential parameter that significantly affects the neural network's performance. Our experiment, as shown in **Fig.4**, demonstrate that an $N_{sl} = 32$ can yield more accurate result.

(3) Overall procedure

Our proposed framework can be concluded as shown in **Fig.5**. Firstly, we train the small-scale neural networks that converts high-dimensional fragments of the structure to low-dimensional ones. This training leads to the acquisition of the downscale part that performs the conversion

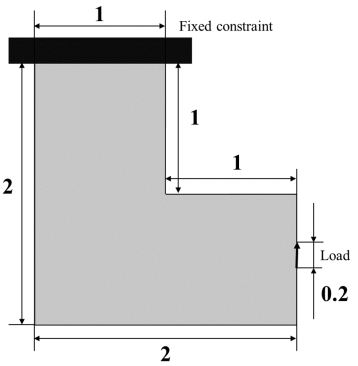


Fig. 6: Design domain and boundary conditions of L-shaped beam

from high-dimensional ($N_{sh} \times N_{sh}$) to low-dimensional ($N_{sl} \times N_{sl}$) fragments, as well as the regeneration part that convert from low-dimensional ($N_{sl} \times N_{sl}$) fragments to high-dimensional ($N_{sh} \times N_{sh}$) ones.

Following the completion of the training phase of pix2pix, we initiate the optimization process for data-driven topology design, leveraging the neural networks trained, as **phase 2** detailed in **Fig.5**. The entire implementation procedure is as follows:

Step1: Evaluate the performance of the high-dimensional initial data by computing the values of the multiple objective functions; **Step2:** Select superior high-dimensional data entities from the high-dimensional evaluated data. Copies of the selected data are stored for merging with the high-dimensional generated data (see **Step9**). We select the higher-performance data based on only the rank-one data entities or on the NSGA-II [8]. **Step3:** Judge whether the high-dimensional elite data satisfy the convergence criterion. **Step4:** Convert the material distributions of the high-dimensional elite data from the shape of the design domain to the shape of the normalized reference domain with $N_h \times N_h$ pixels by the design domain mapping (DDM) [9]. **Step5:** Convert the high-dimensional elite data with $N_h \times N_h$ to low-dimensional ones with $N_l \times N_l$ using downscale part. Here, each fragment of the structure is conversioned as mentioned above. **Step6:** Train VAE using the converted material distributions and newly generate low-dimensional data. **Step7:** Convert the low-dimensional generated data with $N_l \times N_l$ pixels to the high-dimensional data with $N_h \times N_h$ pixels by tiling fragments generated from regeneration part. **Step8:** Convert inversely the generated high-dimensional data from the shape of the reference domain to the shape of design domain by DDM. **Step9:** Evaluate the performances of the high-dimensional generated data, in the same manner as **Step1**. The high-dimensional generated data, including the performance values, are merged with the stored data of **Step2**, and the iteration return to **Step2**.

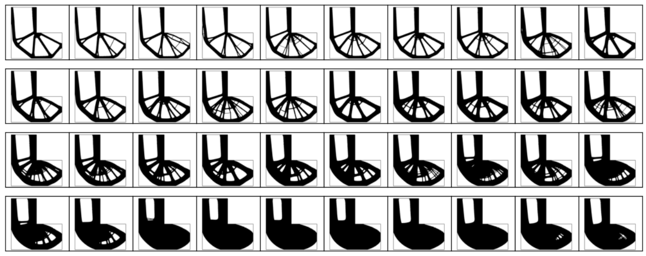
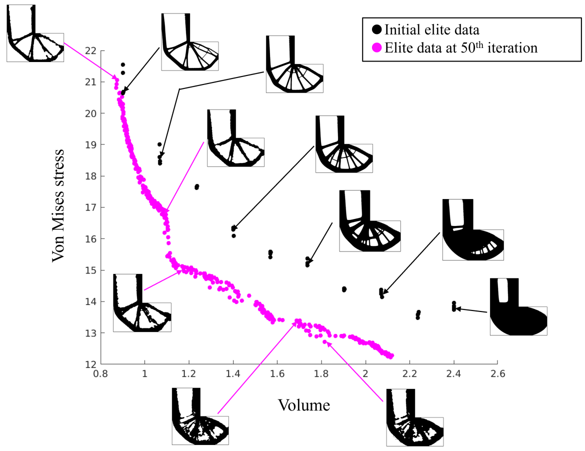
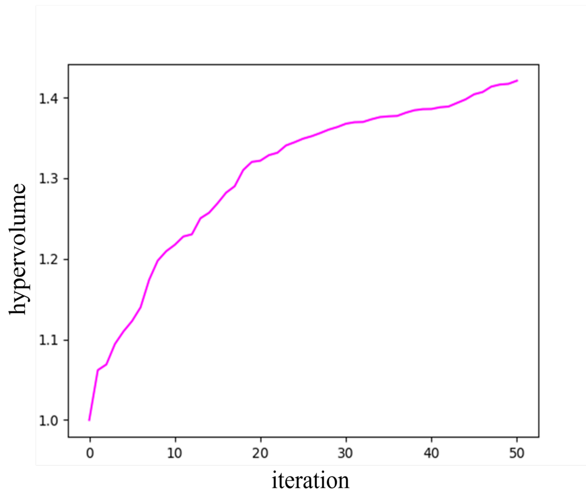


Fig. 7: Initial data for L-shaped topology design



(a) Objective space result



(b) The history of hypervolume indicator

Fig. 8: Results at iteration 50 on the objective space and the hypervolume indicator

4. Numerical example

(1) Problem setting

In the validation phase, we address the L-shaped beam optimization challenge. This requires focusing on a specific design domain, as well as on predefined boundary conditions, both of which are detailed in **Fig.6**. The optimization formulation targets two primary objectives: minimizing the volume and reducing the maximum von Mises stress to the lowest possible level. Additionally, to ensure a mechanical linkage between the displacement-fixed and load-bearing boundaries, we impose a constraint that

Fig. 9: Material distribution with 262,144 dimensions generated by proposed method of the elite data at iteration 50

the mean compliance must not exceed 50. This renders the problem highly nonlinear when approached rigorously, particularly if intermediate states are not permitted. The applied load's magnitude per unit area is standardized at 1.0, with Young's modulus for the structural material also set at 1.0. To prevent a singular stiffness matrix, the modulus is adjusted to 1.0×10^{-3} in void areas, and the Poisson's ratio is maintained at 0.3.

(2) Learning the fragmentation

The validation of fragmentation learning's capability to address high-dimensional challenges, previously unexplored in data-driven topology design, involves discretizing the design domain into $235,200$ square elements. Following the design domain mapping, the dimensionality of the reference domain is effectively translated into 512×512 design variables. By using the methodology mentioned in **section 3**, with the downscale part and regeneration part from generator architecture as depicted in **Fig.3**, this is further reduced to a low-dimensional domain of 128×128 elements to VAE and regenerate the 512×512 data from the newly generated data of VAE. In this dimension, using our approach, the structure of the VAE does not become complex and the previous low-dimensional constructs can still be used.

The entire data-driven topology design process can be carried out by following the steps of **phase 2** mentioned in **section 3-3**. The initial material distribution use 40 optimal structures, as shown in **Fig.7**, obtained by the topology optimization method using the gradient method with different volume constraints and filter radius, with the objective of minimizing the mean compliance.

(3) Optimization attempt

After attempting to compute 50 iterations based on existing hardware, **Fig.8a** presents the results within the objective space, charting both the initial performance and the performance after the 50^{th} iteration as achieved by our proposed methodology. It can be seen that our method does cause the objective function of the data to decrease, indicating that it is optimized in the iterations. Furthermore, **Fig.8b** illustrates the iteration history of the hypervolume indicator, as cited in the work of [2], which is an evaluation method that reflects the amount of elite result generation over the initial solution. As shown in **Fig.8b**, the final outcome at the 50^{th} iteration shows an enhancement compared to the initial performance, with the hypervolume indicator of the proposed method registering an approximate improvement of 42% from its initial value.

Fig.9 shows a dataset consisting of 400 material distributions of the 50^{th} iteration obtained by the proposed method. The obtained material distributions have detailed structural characteristics, indicating that optimal structures with a high degree of design variables can be obtained that can not be computed by the original data-driven topology design. These generated structures by proposed method are reasonable and can go on to be analyzed.

5. Future works

In our study, ”fragmentation learning” emerges as a novel neural network-based strategy for enhancing data-driven topology design. By integrating the pix2pix image transformation neural network and a Variational Autoencoder (VAE), we achieve a significant reduction in the complexity of design variables and can be applied to L-shaped beam problem at the 10^5 level in this work.

However, despite its strengths, our methodology encounters certain limitations that suggest avenues for future research. Initially, the iterative process of our approach requires further validation to ensure its ability to consistently converge towards optimal solutions. Next, the method’s reliance on convolutional layers for dimensional reduction might inadvertently neglect crucial design features, which in turn could compromise the fidelity of reconstructed high-dimensional material distributions by the pix2pix network. Additionally, the pix2pix model’s inherent optimization for square images necessitates a mapping transformation between the reference and design domains, introducing extra complexity and potentially lengthening processing times.

Moreover, the application of our methodology not only contributes to the data-driven topological design of the L-shaped beam design problem, but should also be able to be applied to many other structural problems, which highlights the great potential for future exploration and improvement. It is worth going to note that with the continuous refinement of image transformation models and the continuous improvement of data-driven topology optimization strategies, we can expect to tackle a variety of complex design challenges more efficiently.

6. Conclusion

In summary, the proposed method ”fragmentation learning” based on pix2pix reduces the dimensionality of the design variables handled by VAE and enables the exhaustive generation of candidate solutions under realistic computational costs. This research demonstrates promising capabilities, although we understand their current limitations. This approach facilitates the exploration of solutions for previously intractable high-dimensional challenges, representing a leap forward from conventional topology optimization methods if developed.

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