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A nodal-based Lagrange Multiplier/Cohesive Zone (LM/CZ) Method for Crack Simulations

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In this paper, a novel nodal-based Lagrange Multiplier/Cohesive Zone (LM/CZ) approach for ac- curate crack simulation in quasi-brittle material. Before crack onset, these nodes in the same position are constrained via the Lagrange multipliers (LMs), which need to satisfy the displacement compatibility. And then the constraints are smoothly switched from LMs to cohesive forces once satisfied the transition criterion about the cracking estimation. One numerical example is employed for validation of our approach via comparison with the results from previous researches.

Key Words : *Arbitrary crack paths, Lagrange multiplier method, Cohesive zone model, Coupling method, Artificial compliance.*

1. INTRODUCTION

Over the years, the intrinsic cohesive zone models (CZMs) method always represents a simple yet efficient technique for computational cracking studies [1]. Where the zero-thickness cohesive elements are inserted into the interfaces of finite elements [2], which nodes are constrained via cohesive force before crack onset, and the cohesive forces are calculated based on penalty algorithm. However, that method usually results in the so-called artificial compliance [3]. To deal with this research issue, the Lagrange multipliers (LMs) approach has been recently proposed and effectively coupled with cohesive zone (CZ) approach. The LM/CZ approach has already applied on debonding analysis in laminated composite structures [4]. It is worth to extend the application of the nodal-based LM/CZ approach to simulation of arbitrary crack paths in quasi-brittle structure.

In the conventional nodal-based LM/CZ [3], the constraint enforced in each node consists of only one nodal pair where it connects to. A predictor-corrector form is introduced to calculate LM force at each nodal pair with fulfilling the displacement compatibility [5]. However, to simulate the cracks propagation along unknown path, one finite element node is connected with several nodes and formed several corresponding adjacent nodal pairs. It means that each pair share several nodes with its adjacent pairs. To consider this mutual nodal effect, the constraint force contributed from the adjacent pairs needs to be considered into the LM calculation for the current pair. In this paper, we firstly apply a robust data structure to set these nodes at the same coordinate in one certain group via the linked-list search algorithm [6], which is only once implemented in pre-processing of one numerical case. And then we implement the LM/CZ algorithm in each mutually independent group, where the calculation of LM force in each group is controlled by its error norm less than a specified tolerance. The Gauss-Seidel solution is applied to achieve the end with a limited number of iterations [7].

2. FORMULATIONS

The temporal discretization in this simulation employs the explicit central difference time integration scheme. Herein we assume that the current pair consists of node i and node j. According to the predictor-corrector form of LM method, the position of node i and node j at time t(n+1) in the “predictor” step can be described in Eq. (1),

$$\tilde{\boldsymbol{x}}_{(n+1)} = \boldsymbol{x}_{(n)} + \dot{\boldsymbol{x}}_{(n-1/2)}\Delta t + (\boldsymbol{f}^{\mathrm{ext}}_{(n)} - \boldsymbol{f}^{\mathrm{int}}_{(n)})\Delta t^2/m \qquad (1)$$

where $\tilde{\mathrm{x}}_{(n+1)}$, $\dot{\boldsymbol{x}}_{(n-1/2)}$, $\boldsymbol{f}^{\mathrm{ext}}_{(n)}$ and $\boldsymbol{f}^{\mathrm{int}}_{(n)}$ are the position vector, velocity vector, internal force and external force, respectively. The Eq. (1) is further solved via the improved solution in the “corrector”, as shown in Eq. (2).

$$\boldsymbol{x}_{(n+1)} = \tilde{\boldsymbol{x}}_{(n+1)} + \Delta\boldsymbol{x}_{(n+1)} \qquad (2)$$

with

$$\Delta\boldsymbol{x}_{(n+1)} = (\boldsymbol{\lambda}^{\mathrm{cur}}_{(n)} + \boldsymbol{\lambda}^{\mathrm{adj}}_{(n)})\Delta t^2/m \qquad (3)$$

Where $\boldsymbol{\lambda}^{\mathrm{cur}}_{(n)}$ and $\boldsymbol{\lambda}^{\mathrm{adj}}_{(n)}$ are the Lagrange forces from the current calculated pair and its adjacent pairs, respectively. For the relationship of LMs between two nodes of each nodal pair, we have Eq. (4).

$$\boldsymbol{\lambda}_i^{\mathrm{cur}} = -\boldsymbol{\lambda}_j^{\mathrm{cur}} \tag{4}$$

The Lagrange multiplier of nodal pair needs to satisfy the displacement compatibility condition, as following:

$$\boldsymbol{x}_{i(n+1)} - \boldsymbol{x}_{j(n+1)} = 0 \tag{5}$$

The $\boldsymbol{\lambda}_i^{cur}$ at timestep of (n+1) can be solved by substituting Eqs. (4) and (5) into Eq. (2), as following:

$$\boldsymbol{\lambda}_{i(n+1)}^{cur} = \frac{\tilde{x}_{j(n+1)} - \tilde{x}_{i(n+1)} + (\frac{\lambda_{j(n-1)}^{nbr}}{m_j} - \frac{\lambda_{s(n-1)}^{nbr}}{m_i})\Delta t^2}{(\frac{1}{m_j} + \frac{1}{m_i})\Delta t^2} \tag{6}$$

At the end of the nodal pair loop for LM calculation in each group, the Gauss-Seidel iteration strategy is applied to control the corrector procedure. The error norm L for each group is defined as Eq. (7).

$$\boldsymbol{L} = \sum_{i=1}^{N_p} \| \boldsymbol{g}_i \|_2 < \varepsilon \tag{7}$$

with

$$\boldsymbol{g}_i = \boldsymbol{x}_{i(n+1)} - \boldsymbol{x}_{j(n+1)} \tag{8}$$

where N_p is the total number of LM pairs in the group.

The iteration terminates once the error norm is satisfied. A quadratic stress criterion is then applied to check whether the constraints should switch to cohesive force or not, as shown in Eq. (9).

$$(\frac{<\overline{\boldsymbol{T}}_n>}{T_n^{max}})^2 + (\frac{\overline{\boldsymbol{T}}_t}{T_t^{max}})^2 = 1 \tag{9}$$

Where <·> is Macaulay operator; $\boldsymbol{T}_n^{max}$ and $\boldsymbol{T}_t^{max}$ represent tensile and shear cohesive strengths, respectively; $\overline{\boldsymbol{T}}_n$ and $\overline{\boldsymbol{T}}_t$ represent tensile and shear nodal equivalent stress vector, respectively. $\overline{\boldsymbol{T}}$ can be calculated with the help of LM determined (see Eq. (6)), as the following equation.

$$\overline{\boldsymbol{T}} = \lambda_m \tag{10}$$

Once satisfying the Eq. (9) criterion, the constraint force is described as the cohesive force using a shifted traction-separation law previously utilized in [3].

3. NUMERICAL EXAMPLE

John et al. carried out an experiment to investigate the crack phenomenon in a three-point bending concrete beam. And they also performed the numerical simulation based on the of Linear Elastic Fracture Mechanics (LEFM) [8]. The specimen in [8] is demonstrated in Fig. 1.

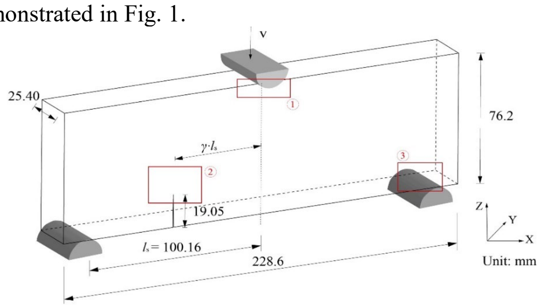

Fig. 1 Three-point bending concrete beam with crack position parameter γ = 0.5.

One can be seen from Fig. 1, there is a center-shifted pre-crack in specimen, and the pre-crack position from the middle of span is determined by a ratio γ, where the γ of 0 represents mode I fracture, while the γ of 0.5 or 0.72 represents the mix-mode fracture. The model was placed on two fixed supports and subjected to a velocity load in the upper support, with a value of 0.050 m/s [9] along the centerline. The beam was discretized using tetrahedral solid elements, with zero-thickness cohesive elements inserted in the element interfaces. The mesh sizes are varied from 1.5 mm to 4 mm, with the refined size of 1.5-2.0 mm in the critical zones, .i.e., near three supports and pre-crack tip, as marked in (numbered in 1, 2 and 3). The concrete beam is assumed to be isotropic elastic, with Young's modulus, mass density and Poisson's ratio of 31.37 GPa, 2400 kg/m^3 and 0.2 respectively. The failure strength of the concrete beam was 3.0 MPa, and the energy release rate was 31.1 N/m [10]. The time history of load curves for the γ = 0.50 case are compared and plotted in Fig. 2.

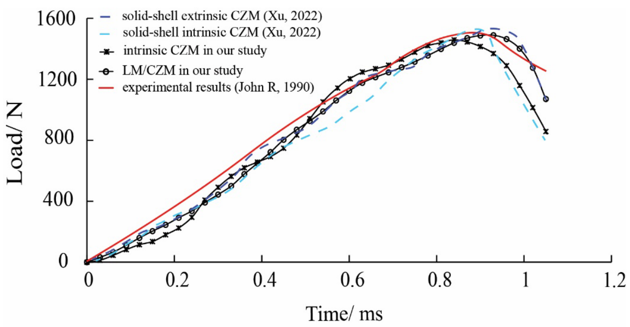

Fig. 2 The time history of load curves (along Z-axis) of three pre-crack from in our study comparing with that from numerical data (Xu et al., 2022) and experimental data (John et al., 1990).

Compared with the curve via intrinsic CZMs approach, a relatively good agreement can be seen in the load curves, and less numerical oscillation can be found in that via LM/CZ approach. In addition, the crack patterns from three notch location cases are illustrated in Fig. 3, where cracks propagate from the notch and gradually upward with a deflection angle from horizontal line.

One can be seen from Fig. 3 that a vertical crack is predicted in the case of γ with 0.0, which represents the fracture mode in pure mode I; and inclined cracks can be found in the rest two cases, where the fracture mode becomes mixed mode I/II. These results from our study are in a good agreement with previous results in three cases, which are not only in the crack propagation and but also in the corresponding deflection angle. These indicate that the LM/CZ can capture the crack behavior of mode I and mix-mode failure, and its effectiveness can also be evaluated quantitatively.

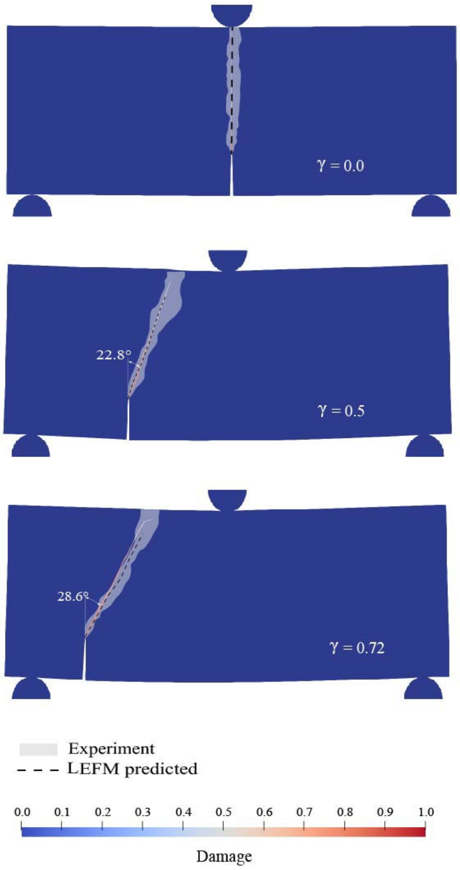

Fig. 3 The predicted cracking patterns of three pre-crack from in our study comparing with that from numerical and experimental data (John et al., 1990).

4. CONCLUSIONS

A novel nodal-based Lagrange Multiplier/Cohesive Zone (LM/CZ) approach is proposed to accurately simulate crack propagation in quasi-brittle material. The predictor-corrector form is applied to calculate the Lagrange multiplier, which is governed by the displacement compatibility in each nodal pair. As cracks may grow in overlapping nodal domains, the Gauss-Seidel iteration strategy is applied to control the error norms due to mutual nodal effects. To validate the accuracy and effectiveness of the method, a numerical example of a concrete beam is employed, and the results obtained from LM/CZ show good agreement with both experimental and numerical data.

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