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Implementing 2D shallow water equations to simulate debris flows using SPH

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This study uses the two-dimensional shallow water equation to govern debris flow behavior after slope failure. In particular, the debris flow is assumed as a homogenous viscous fluid and discretized into depth-integrated particles using the smoothed particle hydrodynamics. This method allows for the tracking of particles in a Lagrangian manner without much consideration of the background mesh. This computational advantage is ideal for wide areas debris flow studies characterized by large computational domains relative to the small percentage of failure areas. Debris flow experiments and actual debris flow disasters are simulated to demonstrate the capability and applicability of the model.

Key Words : *shallow water equations, debris flow, SPH*

1. INTRODUCTION

Debris flow is a common natural geohazard that causes significant damage in mountainous regions. Commonly triggered by earthquakes or rainfall, debris flows often travel over long distances in a fluid-like manner causing destruction along their path. Due to the serious threat it poses to human life, economy, and property, there is a need to better understand the behavior and consequences of this hazard. While the prevention of debris flows remains to be challenging, numerical simulation and modelling are useful tools in minimizing the damages and casualties inflicted on the affected communities.

Therefore, this study employs a smoothed particle hydrodynamics (SPH) approach in discretizing the depth-integrated two-dimensional shallow water equations (2D SWE) to better understand the post-failure behavior, propagation, and extent of multiple debris flows over a wide areas. Particularly, the debris flow is assumed to be a single-phase homogeneous viscous fluid governed by the 2D SWE. The fluid body is discretized using the mesh-free Lagrangian SPH method, whose calculation cost is defined by the number of particles regardless of the extent of the computational domain. Calibration and parameter fitting of the model is conducted to reproduce the debris flow experiments of Iverson et al.[1] and confirm the model's applicability. The calibrated model is then applied on a wide-area scenario to simulate the actual debris flow disasters that occured in Marumori, Miyagi, Japan.

2. NUMERICAL SCHEME

(1) Governing Equations

Neglecting the viscosity terms, the depth-integrated 2D SWE can be written in the Lagrangian form as:

$$\frac{Dh}{Dt} + h\nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

$$h\frac{D\boldsymbol{u}}{Dt} = -\nabla P - gh\nabla z - \frac{\boldsymbol{\tau}_b}{\rho} \tag{2}$$

where D/Dt is the total material deriative, h is the flow depth, $\boldsymbol{u}$ is the velocity vector, $P = \frac{1}{2}gh^2$ is the pressure, g is the acceleration due to gravity, z is the bed elevation, ρ is the density and $\boldsymbol{\tau}_b$ is the bed shear stress vector.

(2) SPH Discretization

The above equations are solved using the mesh-free Lagrangian SPH approach. Under the SPH algorithm, the fluid body is discretized into particles each carrying material properties such as density, pressure, velocity, etc. The approximation of a function is defined using a smoothing kernel function in the integral representation of the function as:

$$< f(\boldsymbol{x}) >= \int_\Omega f(\boldsymbol{x'})W(\boldsymbol{x}-\boldsymbol{x'}, h_s)d\boldsymbol{x'} \tag{3}$$

where $W(\boldsymbol{x}-\boldsymbol{x'}, h_s)$ is the smoothing kernel function, h_s is the smoothing length such that $W = 0$ when $|\boldsymbol{x} - \boldsymbol{x'}| > \kappa h_s$. Here κ is a constant related to h_s that defines the radius of the support domain of the kernel function at point $\boldsymbol{x}$. For brevity, $W(\boldsymbol{x}-\boldsymbol{x'}, h_s)$ is shortened into W_{ij} representing the value of the kernel function between two particles, i and j. This paper employs a cubic B-spline kernel function given by:

$$W_{ij} = \alpha_d \begin{cases} \frac{2}{3} - q^2 + \frac{1}{2}q^3, & 0 \le q < 1 \\ \frac{1}{6}(2-q)^3, & 1 \le q < 2 \\ 0, & q \ge 2 \end{cases} \tag{4}$$

where $\alpha_d = 15/7\pi h_s^2$ is the normalization constant in 2D space, and $q = \frac{|\boldsymbol{x}_i - \boldsymbol{x}_j|}{h_s}$ is defined as the relative distance between particles. Discretizing the integral representation as the summation of N neigboring particles within the support domain of particle i, a function and its gradient can be expressed as:

$$< f(\boldsymbol{x}_i) >= \sum_{j=1}^{N} f(\boldsymbol{x}_j) W_{ij} \Delta \boldsymbol{x}_j \tag{5}$$

$$< \nabla f(\boldsymbol{x}_i) >= \sum_{j=1}^{N} f(\boldsymbol{x}_j) \nabla W_{ij} \Delta \boldsymbol{x}_j \tag{6}$$

where $\Delta \boldsymbol{x}_j$ is the particle's volume in 3D space and the particle's area in 2D space.

Applying the SPH approximation, the continuity and momentum equations can be written as :

$$h_i = \sum_{j=1}^{N} V_j W_{ij} \tag{7}$$

$$\frac{D\boldsymbol{u}_i}{Dt} = -\sum_{j=1}^{N} V_j \left(\frac{P_i}{h_i^2} + \frac{P_j}{h_j^2} + \Pi_{ij} \right) \nabla W_{ij} - \boldsymbol{g}_i - \frac{\boldsymbol{\tau}_{bi}}{\rho h_i} \tag{8}$$

where V_j is the constant volume of a particle, W_{ij} is the kernel function, and Π_{ij} is the stabilizing artificial viscosity term. The vector $\boldsymbol{g}_i$ is defined as $g\nabla z$, where ∇z is calculated from the terrain using finite differences only once and interpolated into the particle positions for simplicity. Note that the 2D SPH area with constant volume $A_j = h_j/V_j$ is analogous with the traditional 3D SPH volume with constant mass $V_j = \rho_j/m_j$.

The artificial viscosity term is used to stabilize and reduce the oscillation in the numerical solution. This paper adopts the artificial viscosity used in the works of Monaghan [2] and Xia [3] as:

$$\Pi_{ij} = -\frac{0.5\nu_{sig} \boldsymbol{u}_{ij} \cdot \boldsymbol{r}_{ij}}{\bar{h}_{ij} |\boldsymbol{r}_{ij}|} \tag{9}$$

$$\nu_{sig} = c_i + c_j - 2\frac{\boldsymbol{u}_{ij} \cdot \boldsymbol{r}_{ij}}{|\boldsymbol{r}_{ij}|} \tag{10}$$

where $\boldsymbol{r}_{ij}$ is the distance vector, $\bar{h}_{ij} = (h_i + h_j)/2$ is the mean flow depth and $c = \sqrt{gh}$ is the wave celerity.

(3) Rheological Model

To simulate debris flow behavior, the flow resistance or bed shear stress term τ_b must to be specified. Particularly, this study employs both the Manning's turbulent model and Coulomb's frictional model defined, respectively, as:

$$\tau_b = \frac{\rho g n^2 u \sqrt{u^2}}{h^{1/3}} \tag{11}$$

$$\tau_b = \rho g h \cos\alpha \tan\phi \tag{12}$$

where n is the Manning's friction coefficient, α is the slope gradient, and ϕ is the internal friction angle of the material. By modifying only two parameters, n and ϕ, the flow characteristics could be modified and calibrated.

3. RESULTS AND DISCUSSION

(1) Debris Flow Flume Experiment

The model is used to simulate the debris flow experiments of Iverson et al. [1]. The experiment involves the release of a debris flow-like mixture of soil and water with a total volume of 11.78 m^3 and wet bulk density of 2,010 kg/m^3 over a rectangular concrete channel 95 m long, 2 m wide, and 1.2 m deep (**Fig. 1**). Upon release of the material, the propagation of the flow front is measured and plotted with time.

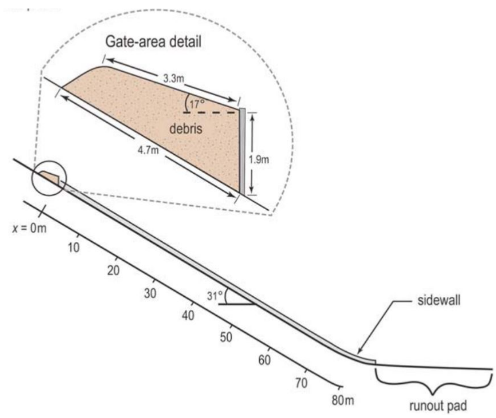

Fig. 1 Flume experimental setup, taken from Iverson et al.

To simulate the experiment, the debris flow was discretized into 2,353 particles with initial spacing of $\Delta x = 0.1$ m. The radius of the support domain is further defined as $\kappa h_s = 2.4\Delta x$. To maintain a reasonable amount of neighboring particles during the flow expansion or contraction, the smoothing length is modified and updated with respect to the initial depth and smoothing length of each particle. Wall boundary conditions are implemented using fixed virtual fluid particles whose positions and velocities are not evolved through time.

Parameter fitting and calibration of the flow resistance parameters, n and ϕ, are conducted to reproduce the results of the experiment. **Fig. 2** shows the effect of varying the flow resistance parameters in modifying the flow behavior. In both instances, reducing the values leads to less flow resistance and therefore higher flow velocities. The parameters that best replicated the experimental results are Manning's friction coefficient $n = 0.03$ for the flume floor

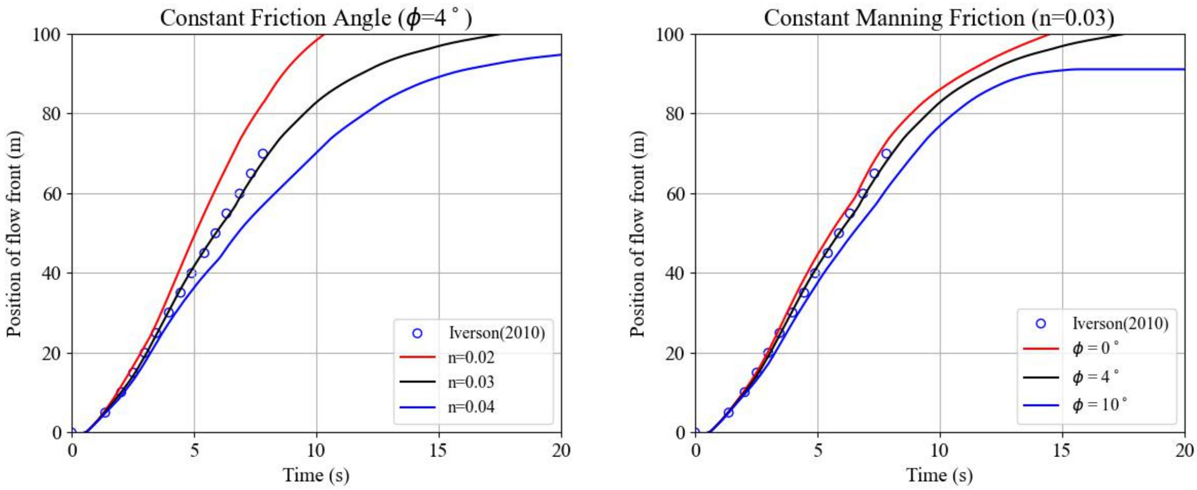

Fig. 2 Graphs showing the propagaton of the experimental and simulated flow front in time. The left figure shows the effect of varying the friction coefficient, while the right shows the effect of varying the internal friction angle.

and an internal friction angle $\phi = 4°$. This friction coefficient value corresponds to rough concrete or gravel bottom ($n = 0.03$) [4] consistent with the conditions of the experimental setup, while this friction angle suggests the residual strength of a post-failure mixture of fluidized soil and water. Sufficient selection of the parameters shows that the model can result in a reasonable agreement with the experiment demonstrating the applicability of both the numerical and rheologic model.

(2) Marumori Debris Flow Disasters

The model is then applied to simulate the debris flow disasters that occurred during the 2019 Typhoon Hagibis in Marumori, Miyagi, Japan. The terrain is characterized using a high-resolution digital elevation model (1 meter) of the study area taken after the 2019 event. The distribution of the source of slope failure is inferred from the analysis of post-event terrain morphology and the delineation of the actual observable debris flows. The geometric features of the slope failure is extracted from the post-failure condition of the terrain. The elevations (ridges) and depressions (valleys) are highlighted using a neighborhood focal statistics algorithm, where the mean elevation of the neighboring cells is subtracted from the elevation of the focal cell. A negative difference value specifies that the focal cell is lower than its neighbors and can then be considered a depression, and vice versa.

A threshold difference value is then determined using the delineated actual failure geometry. Moreover, assuming that the terrain is the equilibrated condition after slope failure, a threshold slope angle could be employed to highlight areas whose gradients have been lowered after failure. The intersection of the above conditions is then manually post-processed to remove unwanted artifacts outside the delineated failure geometry. Finally, the depths (volumes) are assumed to be the difference values obtained in

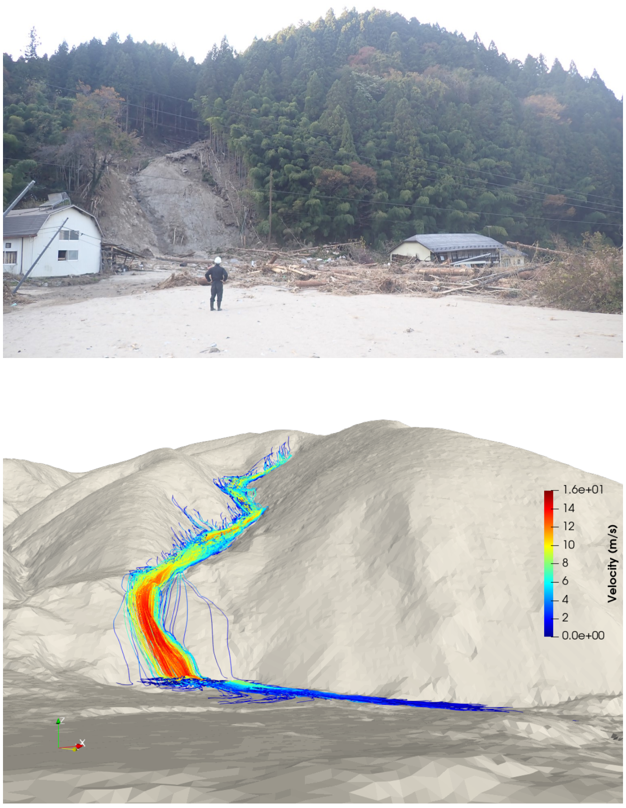

Fig. 3 Actual and simulated debris flow within Usudaira community.

the earlier neighborhood focal statistics algorithm.

Taking the experimental results earlier to be a quantitative benchmark of debris flow behavior, the Marumori debris flows are simulated using the same material properties and calibrated parameters in section 3.1, The Manning's friction coefficient, however, was modified to n=0.07 to

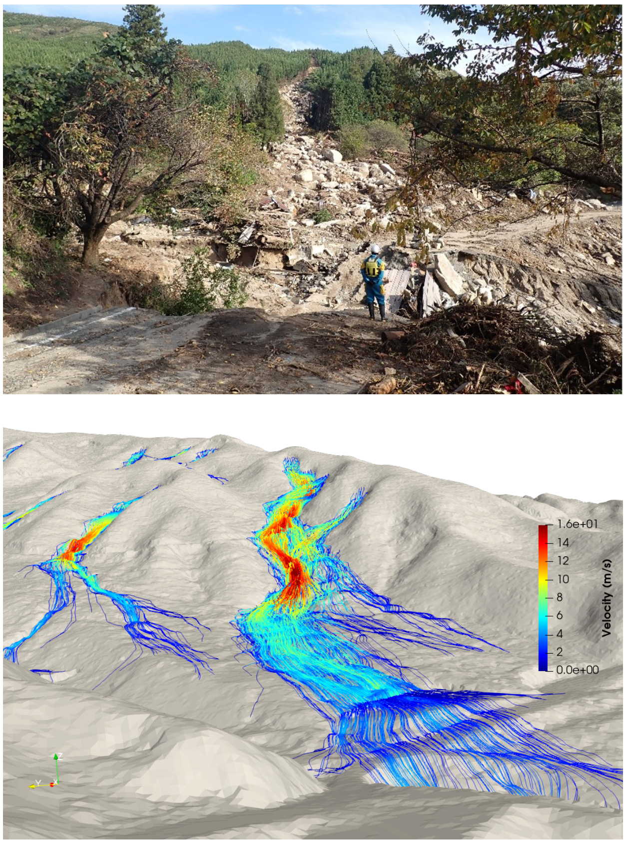

Fig. 4 Actual and simulated debris flow within Koyasu community.

correspond to mountain streams lined with cobbles and boulders [4] resembling actual field conditions.

Two areas in Marumori that sustained significant damage during the typhoon are analyzed in detail. The first area is the Usudaira community adjacent to the Gofukuya River that suffered multiple flooding and slope disasters. The computational domain covers a total area of 312,477 m^2, with debris positioned at the source areas and discretized into 17,608 particles with initial spacing of $\Delta x = 0.5$ m. The next area is within the Koyasu community covering a compuational domain area of 740,911 m^2. The debris flow along the source areas is dicretized into 32,292 particles with initial spacing of $\Delta x = 0.5$ m.

Fig. 3 and **Fig. 4** show some of the results within the computational domain of Usudaira and Koyasu communities, respectively, each depicting a simulation of a single debris flow event after 60s. It illustrates the pathlines of individual debris flow particles visualized (masked) at 20 particle intervals for better clarity. The pathlines depict the evolution of particle positions and velocities through space and time.

The results show peak velocities of 16.2 m/s and 17.4 m/s for Usudaira and Koyasu, respectively, conforming with local and global debris flow velocity estimates. Moreover, particle tracking also provides information regarding total distance travelled reaching distances of up to 600 m for both areas. An aerial view of the distribution of pathlines suggest that the model generally captures the physical behavior and response of the debris flows as confirmed by the close agreement of the simulated debris flow distributions with the delineated damaged areas (**Fig. 5**).

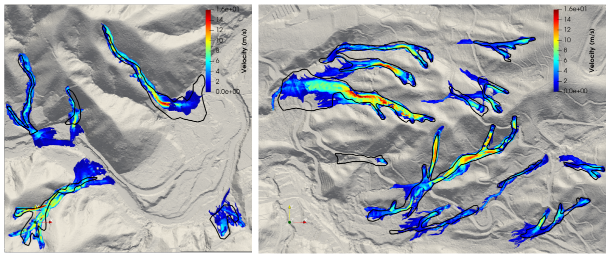

Fig. 5 Simulated results overlain by actual damage.

The above simulations demonstrate the applicability of the model in characterizing multiple debris flow occurrences over wide areas with intuitive and reasonable results. The model could provide necessary information such as debris flow velocities, propagation direction, and runout distributions that are expected to aid in the consideration, placement, and design of both soft and hard disaster prevention measures.

4. CONCLUSIONS

Using an SPH-based approach in solving the depth-integrated shallow water equations, the propagation velocity, behavior, and extent of an idealized single-phase debris flow have been modeled. Calibration of two material parameters related to the flow resistance terms allowed the model to conform with past or experimental data. Despite the numerical and theoretical simplifications, proposed method successfully reproduced the flow velocities of the full-scale experiment of Iverson et al.[1] and the runout extent of the debris flow disasters that occurred in Japan. Therefore, given proper calibration, the model can be used in the evaluation, assessment, or mitigation of would-be future debris flow hazards in the area. Moreover, the simplified 2D SWE framework provides a practical alternative for wide-area debris flow hazard studies.

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