An evolutionary approach to the Bertrand paradox in finitely repeated games*

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Abstract

Finite-horizon price competition models with discrete price spaces present a challenge for standard equilibrium analyses, as uniquely predicted paths often fail to capture stability against strategic deviations. Here we introduce neutrally stable strategies (NSS) as a refinement that evaluates stability based on deviation deterrence rather than subgame-perfect rationality. We show that, in finitely repeated price competition with a unique stage-game equilibrium, NSS yields a class of price paths that satisfy one-shot deviation deterrence conditions in every period. Crucially, while the required continuation payoff in each period is uniquely determined by these conditions, the specific price sequences that realize these payoffs are generally non-unique. We operationalize this insight by constructing explicit terminal phases that implement the deviation-deterrence bounds and illustrate how continuation values can be allocated recursively even in a finite horizon. Our results reveal a structural multiplicity akin to a folk theorem, demonstrating that evolutionary stability concepts can support a wide range of stable price dynamics beyond what subgame-perfect Nash equilibrium predicts.

JEL classification: C72, C73

Keywords: Price competition, Bertrand paradox, Folk theorem.

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