The Weighted Margin Rules and Condorcetian Criteria

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Abstract

This paper analyzes the axiomatic properties of margin-based voting rules with a focus on Condorcetian criteria. We first examine the minimax rule, which selects alternatives minimizing the maximum pairwise loss. While the minimax rule satisfies the Condorcet loser criterion when there are exactly three alternatives, we demonstrate that it violates the criterion in cases with four or more alternatives.

To generalize our analysis, we introduce a parametric class of mechanisms called Weighted Margin Rules (WMR). A WMR evaluates alternatives based on a linear combination of their ordered pairwise margins, where the weights are defined on an $(m-2)$-dimensional simplex. This framework encompasses various classical rules, including the minimax and Borda rules, as specific instances of weight distributions. Our main theorem provides a characterization of the minimax rule: within the entire WMR family, it is the unique rule that satisfies the Condorcet winner criterion for any number of alternatives $m \geq 3$ and voters $n \geq 3$. These results clarify the geometric structure of margin-based rules and establish that focusing on the worst-case pairwise margin is a necessary condition for Condorcet winner consistency in this broad class of rules.