A Hybrid Support Enumeration Algorithm for Efficiently Finding All Nash Equilibria in N-Player Games

Ao Liu[*]   Yohei Mitani[†]

January 30, 2026

Abstract

Computing the full set of Nash equilibria in finite normal-form games is central to many economic applications but remains computationally prohibitive in general, as exhaustive support enumeration scales exponentially with the number of players and actions. This paper proposes a hybrid support-enumeration solver that preserves completeness under standard nondegeneracy while substantially reducing wasted computation. The method combines an ordered pruning pipeline of theoretically safe certificates—ranging from support-structure restrictions and dominance-based eliminations to linear and convex feasibility tests—to discard most candidate supports before any root-finding is attempted. For the remaining supports, we route each induced indifference system to a specialized backend: linear algebra when exactly two players mix; precompiled elimination-template solvers for small, frequently recurring polynomial structures generated offline; and multihomogeneous homotopy continuation as a robust fallback for larger or untemplated cases. We provide a correctness result showing that, under nondegeneracy and safe pruning, the algorithm returns exactly the set of Nash equilibria after verification against probability constraints and best-response inequalities. Computational experiments on random benchmark games show that elimination templates are consistently faster than homotopy continuation on fully mixed instances, and that the full hybrid solver substantially outperforms existing enumeration baselines across a range of game sizes.

Keywords: Nash equilibrium; support enumeration; elimination templates; homotopy continuation.

---

[*]Graduate School of Agriculture, Kyoto University. Email: `liu.ao.72w@st.kyoto-u.ac.jp`
[†]Graduate School of Agriculture, Kyoto University. Email: `mitani.yohei.7w@kyoto-u.ac.jp`